Universität Mannheim
Lehrstuhl für Mathematik VI

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Exercises Algebraic Geometry
Sheet 1

1. Let k be a field, $I_1, I_2 \subset k[x_1, \ldots, x_n]$ ideals and X_1, X_2 affin-algebraic sets in k^n.
   (a) Show that $I_1 \subset I_2$ implies $V(I_1) \supset V(I_2)$.
   (b) Suppose $X_1 \subset X_2$. Show that $I(X_2) \subset I(X_1)$.
   (c) Show that $V(I_1 \cap I_2) = V(I_1 \cdot I_2) = V(I_1) \cup V(I_2)$. Give an example where $I_1 \cdot I_2 \subsetneqq I_1 \cap I_2$.
   (d) Show that $V(I_1 + I_2) = V(I_1) \cap V(I_2)$.
2. Let R be Noetherian and A be a finitely generated R-algebra. Show that A is also Noetherian. (hint: study inclusion of ideals in factor rings)
3. Prove the following version of Hilbert's Nullstellensatz: Any maximal ideal $\mathbf{m}$ in the polynomial ring $k[x_1, \ldots, x_n]$ over an algebraically closed field k is of the form $\mathbf{m} = (x_1 - a_1, \ldots, x_n - a_n)$ for some $(a_1, \ldots, a_n) \in k^n$.
4. Give an heuristic argument why the number of lines on a cubic surface $S = V(P) \subset \mathbb{C}^3$ (with $P \in \mathbb{C}[x, y, z]_{\leq 3}$) can be expected to be finite. (hint: Count the number of parameters for a line in $\mathbb{C}^3$ and express the condition of being contained in a cubic)
5. Draw pictures of the following curves $C = V(f)$ (in an appropriate neighborhood of the origin). You may use the program *surf* (available via SourceForge) to do it, but you should at least try to "figure out" how to obtain these pictures (or better draw them by hand)!
   (a) $f = x^2 - y^2 \in \mathbb{R}[x, y]$.
   (b) $f = x^2 + y^2 \in \mathbb{R}[x, y]$.
   (c) $f = y^2 - (x^2 - 1) \cdot (x^2 - 4) \cdot (x^2 - 9) \in \mathbb{R}[x, y]$.
   (d) $f = y^2 - x^3 + x^2 \in \mathbb{R}[x, y]$.
   (e) $f = y^2 - (x^2 - 1) \cdot (x^2 - 4)^2 \in \mathbb{R}[x, y]$.

Alle Informationen zur Vorlesung (Termine, Übungsblätter, Skript etc.) sind unter

`http://hilbert.math.uni-mannheim.de/~sevenhec/AlgGeom07.html`

zu finden.