

Programming with Nonequispaced FFT

Solution 1

C Library Hands On

Exercise 1:

The routine `simple_test_nfft_1d()` initialises a plan for a one-dimensional nfft with $N = 14$ Fourier coefficients and $M = 19$ nodes, generates random nodes $x_j \in [-\frac{1}{2}, \frac{1}{2}]$, $j = 0, \dots, 18$, and precomputes the matrix $\mathbf{C} \in \mathbb{R}^{19 \times 32}$. Random Fourier coefficients $\hat{f}_k \in \mathbb{C}$, $k = -7, \dots, 6$ are generated as well and the trigonometric polynomial $f(x) = \sum_{k=-7}^6 \hat{f}_k e^{-2\pi i k x}$ is evaluated at the nodes x_j by the routine `ndft_trafo` (direct way), and by `nfft_trafo` using the approximation scheme as outlined in the lecture. Coefficients and function values are displayed. Also, the adjoint transforms are executed - clearly showing that the NFFT is, in contrast to the FFT, a non-unitary transform.

The output on JuDGE looks like:

```
1) computing a one dimensional ndft, nfft and an adjoint nfft

given Fourier coefficients, vector f_hat, adr=0x603130
 0. +3.0E-01+5.3E-01i,+9.2E-01+8.1E-01i,+1.9E-01+5.7E-01i,+7.7E-02+9.8E-01i,
 4. +1.2E-01+7.8E-01i,+1.0E-01+2.0E-02i,+3.8E-01+6.8E-01i,+7.5E-01+6.2E-01i,
 8. +1.3E-01+7.7E-01i,+1.9E-01+5.1E-01i,+3.2E-01+3.7E-01i,+8.8E-01+5.3E-01i,
12. +4.3E-01+6.7E-01i,+1.7E-01+8.6E-01i,

ndft, vector f, adr=0x603220
 0. +1.2E+00-3.8E-02i,+1.3E+00-3.4E-02i,+1.2E+00-5.3E-01i,+2.3E+00+1.2E+00i,
 4. -2.8E-01+6.3E-02i,-3.6E-01-1.8E+00i,-3.7E-01-1.8E+00i,+5.0E+00+8.1E+00i,
 8. +3.1E+00+6.8E+00i,+2.7E+00+3.6E+00i,-2.1E-01+8.2E-01i,-3.2E-01+8.6E-01i,
12. -3.7E-01-8.9E-01i,-5.7E-01-8.8E-01i,+2.4E-01+1.3E+00i,+8.9E-01-7.9E-01i,
16. +1.1E+00-6.7E-01i,+1.5E+00-1.0E-01i,-8.5E-01-8.1E-02i,
took 3.696767e-05 seconds.

nfft, vector f, adr=0x603220
 0. +1.2E+00-3.8E-02i,+1.3E+00-3.4E-02i,+1.2E+00-5.3E-01i,+2.3E+00+1.2E+00i,
 4. -2.8E-01+6.3E-02i,-3.6E-01-1.8E+00i,-3.7E-01-1.8E+00i,+5.0E+00+8.1E+00i,
 8. +3.1E+00+6.8E+00i,+2.7E+00+3.6E+00i,-2.1E-01+8.2E-01i,-3.2E-01+8.6E-01i,
12. -3.7E-01-8.9E-01i,-5.7E-01-8.8E-01i,+2.4E-01+1.3E+00i,+8.9E-01-7.9E-01i,
16. +1.1E+00-6.7E-01i,+1.5E+00-1.0E-01i,-8.5E-01-8.1E-02i,

adjoint ndft, vector f_hat, adr=0x603130
 0. -2.1E-01+1.4E+01i,+1.3E+01+1.6E+01i,-1.3E-01+9.6E+00i,+8.1E+00+1.8E+01i,
 4. +1.1E+01+2.5E+01i,+1.0E+01+1.2E+01i,+4.0E+00+2.4E+01i,+1.7E+01+1.5E+01i,
 8. +4.4E+00+2.2E+01i,+1.8E+01+1.5E+01i,+2.3E+00+1.4E+01i,+1.9E+01+1.4E+01i,
12. +1.1E+00+5.3E+00i,+1.2E+01+1.8E+01i,

adjoint nfft, vector f_hat, adr=0x603130
 0. -2.1E-01+1.4E+01i,+1.3E+01+1.6E+01i,-1.3E-01+9.6E+00i,+8.1E+00+1.8E+01i,
 4. +1.1E+01+2.5E+01i,+1.0E+01+1.2E+01i,+4.0E+00+2.4E+01i,+1.7E+01+1.5E+01i,
 8. +4.4E+00+2.2E+01i,+1.8E+01+1.5E+01i,+2.3E+00+1.4E+01i,+1.9E+01+1.4E+01i,
12. +1.1E+00+5.3E+00i,+1.2E+01+1.8E+01i,
```

Exercise 2:

The initialisation with no precomputation of \mathbf{C} , compressed storage, and explicit storage are given by (line)

```
nfft_init_guru(&p, 2, N, M, n, 7,  
              PRE_PHI_HUT| MALLOC_F_HAT| MALLOC_X| MALLOC_F |  
              FFTW_INIT| FFT_OUT_OF_PLACE,  
              FFTW_ESTIMATE| FFTW_DESTROY_INPUT);
```

```
nfft_init_guru(&p, 2, N, M, n, 7,  
              PRE_PHI_HUT| PRE_PSI| MALLOC_F_HAT| MALLOC_X| MALLOC_F |  
              FFTW_INIT| FFT_OUT_OF_PLACE,  
              FFTW_ESTIMATE| FFTW_DESTROY_INPUT);
```

and

```
nfft_init_guru(&p, 2, N, M, n, 7,  
              PRE_PHI_HUT| PRE_FULL_PSI| MALLOC_F_HAT| MALLOC_X| MALLOC_F |  
              FFTW_INIT| FFT_OUT_OF_PLACE,  
              FFTW_ESTIMATE| FFTW_DESTROY_INPUT);
```

For a nonequispaced Fourier transform with $N_0 = 70$ and $N_1 = 90$, i.e. $N_0N_1 = 6300$ Fourier coefficients, FFT mesh sizes $n_0 = 140$, $n_1 = 180$ and $M = 25200$ evaluation nodes, the computation times are similar to the following.

Results on JuDGE:

transform	cpu time (secs.)
ndft	3.265577e+00
nfft, no precomputation	2.076254e-02
nfft, PRE_PSI	6.812286e-03
nfft, PRE_FULL_PSI	7.821936e-03