Preconditioners for ill–conditioned Toeplitz matrices

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Abstract. This paper is concerned with the solution of systems of linear equations $\boldsymbol{A}_N \boldsymbol{x} = \boldsymbol{b}$, where $\{\boldsymbol{A}_N\}_{N\in\mathbb{N}}$ denotes a sequence of positive definite Hermitian ill–conditioned Toeplitz matrices arising from a (real–valued) nonnegative generating function $f \in C_{2\pi}$ with zeros. We construct positive definite Hermitian preconditioners $\boldsymbol{M}_N$ such that the eigenvalues of $\boldsymbol{M}_N^{-1}\boldsymbol{A}_N$ are clustered at 1 and the corresponding PCG–method requires only $\mathcal{O}(N \log N)$ arithmetical operations.
We sketch how our preconditioning technique can be extended to symmetric Toeplitz systems, doubly symmetric block Toeplitz systems with Toeplitz blocks and non–Hermitian Toeplitz systems.
Numerical tests confirm the theoretical expectations.

1991 *Mathematics Subject Classification.* 65F10, 65F15, 65T10.
Key words and phrases. Ill–conditioned Toeplitz matrices , CG–method, clusters of eigenvalues, preconditioners.