Surjectivity of linear partial differential operators on spaces of scalar valued and vector valued distributions

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The question of solvability of a linear partial differential equation with constant coefficients $P(D)u = f$ in some open set $X \subseteq \mathbb{R}^d$ is a classical problem in mathematical analysis. Depending on the properties of the right hand side f this problem leads in a natural way to the question of surjectivity of $P(D)$ on various spaces of functions and distributions.

By a result of Malgrange, surjectivity of $P(D)$ on $C^\infty(X)$ is equivalent to surjectivity on local Sobolev spaces $H^{s,\mathrm{loc}}(X)$, and this is characterized by a combined property of X and $P(D)$ called P-convexity for supports of X. However, this property is not enough to ensure surjectivity of $P(D)$ on $\mathscr{D}'(X)$ in general. It was proved by Hörmander that the latter is true if and only if X is P-convex for supports and, additionally, P-convex for singular supports. Although these results have been proved more than 50 years ago, there are still only very few classes of differential operators for which the P-convexity conditions can be evaluated in concrete situations.

We will discuss the P-convexity conditions and present some results leading to a more accessible geometric characterization of surjectivity for certain classes of operators.

Moreover, we consider the question whether for a surjective differential operator $P(D)$ the equation $P(D)u_\lambda = f_\lambda$ is solvable in such a way that if f_λ depends "nicely" (e.g. holomorphically) on the parameter λ, then the solution u_λ can be chosen depending on λ in the same way. The problem of parameter dependence of solutions will lead us to consider $P(D)$ on vector valued distributions.