# Low-ranks in computational Fourier analysis

Stefan Kunis (Universität Osnabrück)

Sparsity and low rank structures have gained a lot of attention in mathematics ever since. In this series of lectures, we discuss two specific scenarios in computational Fourier analysis.

At first, we address the construction of fast algorithms by exploiting local low ranks. While classical fast algorithms rely on structural properties of specific problems, more general problems would not allow for a fast algorithm in infinite precision but often do so when considered for finite accuracy. We discuss the idea of such schemes which uses low rank approximations under certain admissibility conditions and dedicated divide and conquer strategies. Examples are given for specific discretised integral operators including so-called hierarchical matrices for asymptotically smooth kernel functions and butterfly schemes for oscillatory kernels. In particular, this leads to a fast Fourier transform for sparse data with an application in photoacoustic tomography and to a fast evaluation scheme for polynomials in the unit disc.

Secondly, we discuss the reconstruction of sparse signals from given Fourier data. We will shortly review the results in “compressed sensing” where the signal is a sparse vector of finite length and proceed with the situation that the signal is a finitely supported complex measure on the unit interval. The reconstruction of this measure from its low frequency Fourier coefficients can be accomplished by Prony’s method and its variants. We review the established methods including semidefinite program formulations as discussed also in the lecture series of Jan Vybiral. Finally, the generalisation to the multivariate case, i.e., the signal is a finitely supported complex measure on the d-dimensional unit cube, leads to several interesting algebraic and algorithmic questions which we could solve recently.