Vector-valued Versions of the Logarithmic Residue Theorem

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The logarithmic residue theorem from complex function theory states that the value of the contour integral of the logarithmic derivative $f'f^{-1}$ of a scalar analytic function f is a nonnegative integer (equal to the total number of zeros of f inside the contour). The talk is concerned with generalizations of this fundamental result to the situation where f has its vales in a (complex) Banach algebra $\mathcal{B}$. Among the situations discussed are:

- the Banach algebra $\mathcal{B}$ is equal to the full matrix algebra $\mathbb{C}^{n\times n}$;
- the Banach algebra $\mathcal{B}$ is a subalgebra of the matrix algebra $\mathbb{C}^{n\times n}$ determined by a reflexive and transitive pattern of zeros;
- the values of f are Fredholm operators on a Banach space X;
- the Banach algebra $\mathcal{B}$ is the operator algebra generated by the compact operators and the identity operator on a Banach space X;
- the Banach algebra $\mathcal{B}$ is a C^*-algebra;
- the function f has commuting values.

Properties of the set of vector-valued logarithmic residues – i.e., the contour integrals of vector-valued logarithmic derivatives – will be considered too. There is a close connection with the issue of sums of idempotents.

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