Spectral Regularity
and
Sums of Idempotents in Banach Algebras

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A basic result from complex function theory states that the contour integral of the logarithmic derivative of a scalar analytic function can only vanish when the function has no zeros inside the contour (so takes invertible values in $\mathbb{C}$ there). The principal issue dealt with here is to what extent this result generalizes to the vector-valued case where the underlying Banach algebra is different from $\mathbb{C}$ (possibly infinite dimensional). In case it does, the Banach algebra in question is called *spectrally regular*. There are important Banach which are spectrally regular, but there are also non-exotic Banach algebras failing to have this property.

Positive results have been obtained for matrix algebras and situations that are somehow close to the matrix case. Here certain rank/trace considerations are instrumental. Commutative Banach algebras are spectrally regular too. In generalizations of this result – for instance to polynomial identity algebras – non-commutative Gelfand theory, involving families of matrix representations, plays an essential role.

Among the (vector-valued) contour integrals meant above are all *sums of idempotents*. Spectral regularity implies that there are no non-trivial zero sums of idempotents. All known instances of Banach algebras failing to be spectrally regular have been found via the construction of five non-zero idempotents adding up to zero, and it is a striking fact that one cannot make do with less than five. The idempotent constructions touch on deep problems concerning the geometry of Banach spaces and general topology. In this context a novel way to produce Cantor sets has been developed.

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