

# An opinionated peek into approximation theory and orthogonal polynomials

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## (Some) Inequalities in Approximation Theory

I will discuss polynomial inequalities associated with the names of A. Markov, Bernstein, Schur, and Remez, and their extraordinary but not fully unexpected impact on certain fundamental problems in approximation theory and orthogonal polynomials. I will also give convincing reasons why at least two of these inequalities are unfairly and incorrectly referred to as they are by those who are not as much sticklers for the truth as I've been trying to be lately.

PUZZLE. Who of the above are “Russian” and, if Leibniz were right proclaiming that we have “die beste aller moeglichen Welten”, which “Hungarians” would replace them?

## The “ $(x_n)$ ” sequence

I will discuss my beloved “ $(x_n)$ ” sequence, or, in other words, the recurrence coefficients for the orthogonal polynomials associated with the weight function  $\exp(-c/4x^4 - K/2x^2)$  on the real line that have some fascinating properties that have long thrilled and puzzled me. In certain fancy circles the equation describing this sequence is called Discrete Painlevé Equation (googleable) but in our down to earth universe it is just a non-linear second order difference equation with mystic but irresistible beauty and attraction.

PUZZLE. What is the difference between an engineer, a physicist, and a mathematician?

## Christoffel Functions

I will discuss the primary reasons why I fell in love with Christoffel Functions (googleable) and, despite my unrelenting love, why I stopped studying them.

PUZZLE. Who and when coined the expression “Christoffel Function”? Neither Shohat (1933) nor Erdős–Turán (1941) use this terminology despite explicitly studying them. The term “Christoffel Number” was already used before 1940.