Matrix completion problems

Charles Johnson (College of William & Mary, USA)

A partial matrix is a rectangular array, in which some entries are specified, while others are free to be chosen from and agreed upon set (eg the real numbers). A completion of a partial matrix is a choice of values for the unspecified entries that results in a conventional matrix. A matrix completion problem asks which partial matrices have some completion of a desired type (eg positive definite). Matrix completion problems arise in a variety of ways and have now been studied for 30+ years. They also provide a way to get further insight into important classes of matrices. In some cases, optimization of a matrix parameter, such as the determinant, among completions of a given type is of interest. We survey the work on matrix completion problems, both historical and recent. Of particular interest will be positive definite completions, totally positive completions, minimum rank completions, but many more as well. We describe completion techniques, the relationship with semi-algebraic sets, the structure of determinant maximizing completions, and connections with other mathematical ideas, such as Bruhat order, etc.