

Abstract

Fractal algebras of discretization sequences

Steffen Roch (TU Darmstadt)

First a warning: Fractality, in the sense of these lectures, has nothing to do with fractal geometries or broken dimensions or other involved things. Rather, the notion *fractal algebra* had been chosen in order to emphasize an important property of many discretization sequences, namely their *self-similarity*, in the sense that each subsequence has the same properties as the full sequence. (But note that self-similarity is also a characteristic aspect of many fractal sets. I guess that everyone is fascinated by zooming into the Mandelbrot set, which reveals the same details at finer and finer levels.)

We start with a precise definition of the concept of fractality and show that the fractal property is enormously useful for several spectral approximation problems. These results will be illustrated by sequences in the algebra of the finite sections method for Toeplitz operators. (*What else?* one might ask: these algebras (first) played the prominent role in the development of the use of algebraic techniques in numerical analysis, and they were (second) a main object of study in Silbermann's school; so one can hardly think of a lecture on this topic in Chemnitz, which does not come across with these algebras.)

Then we discuss some structural consequences of fractality, which are related with the notion of a compact sequence. Discretized Cuntz algebras will show that the idea of fractality is also a very helpful guide in order to analyse concrete algebras of approximation sequences, which illustrates the importance of the idea of *fractal restriction*. Our final example is the algebra of the finite sections method for band operators. This algebra is not fractal, but has a related property which we call *essential fractality* and which is related with the approximation of points in the essential spectrum.

I suppose that the participants have some (really) basic knowledge on C^* -algebras and their representations. A short script will be available during the Summer School.