Abstract

Spectra and finite sections of band operators

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We are looking at bounded linear operators $A$ on scalar- or vector-valued $\ell^p$ spaces. Our operators are given by infinite (in general non-hermitian) matrices with finitely many nonzero diagonals. Studying the concept of a so-called limit operator, we will give a formula for the essential spectrum of $A$ that can be made concrete in the case of certain types (random, almost periodic, slowly oscillating) of diagonals. For the case of three diagonals we also have upper bounds on spectrum and pseudospectrum. These upper bounds are derived by looking at certain submatrices of $A$ with finite size $n$, and they converge to the (pseudo)spectrum of $A$ if $n \to \infty$.

In a second thread we will discuss the stability of truncation methods – versions of the so-called finite section method – for the approximate solution of equations $Ax = b$. The main idea is, again, to cut finite matrices out of the infinite matrix $A$ and to solve the corresponding finite systems. We learn how to utilize the freedom of choice of those submatrices from a bi-infinite matrix in order to end up with a convergent approximation method.