Orthogonal polynomials and numerical methods for integral equations

The topics of this series of lectures and exercises are the following:

1. Basic properties of orthogonal polynomials
   (a) Chebyshev and Jacobi polynomials
   (b) Interpolation and quadrature processes based on the zeros of orthogonal polynomials
   (c) Mapping properties (described by orthogonal polynomials) of singular and hypersingular integral operators
2. Stability of collocation methods for singular integral equations
   (a) Basics from Functional Analysis
   (b) A concept for the proof of the convergence of numerical methods for operator equations
   (c) Application to collocation methods (based on the zeros of orthogonal polynomials) for integral equations
3. Design of fast algorithms
   (a) Discussion of the structure of the discretized equations
   (b) Two- or multiple grid methods
   (c) Application of discrete trigonometric transforms and Krylov subspace methods

As examples we could consider the numerical solution of the generalized airfoil equation, of a nonlinear integral equation of Prandtl's type from fracture mechanics and of nonlinear Cauchy singular integral equations arizing from certain two-dimensional free boundary value problems.

Amount: 3–5 lectures, 2–3 exercises (each 1–1,5 hour)