The B-model Frobenius manifold

$R^\vee = \mathcal{E} \otimes_{\mathbb{C}[\hbar]} \mathbb{C}[\hbar^{\pm 1}]$

$\cup\!\!\shortmid$

We have $(\mathcal{E}, \nabla, (\ ,\ ))$ $\quad \mathcal{E} \longrightarrow \mathcal{H} = \{\nabla s = 0\}$

$(\mathcal{H}$ free $\mathbb{C}\{\hbar^{\pm 1}\}$-module$)$, find basis

$[\Omega] \equiv S_0^B \mod \mathcal{H}_-$

$x = 0 \in \mathcal{M}$

"opposite subspace" $\quad t_0 = t_1 = \dots = t_n = 0$

$\mathcal{H} = \mathcal{H}_- \oplus \mathcal{E}(x)$.

$\frac{\infty}{2}$-VHS $+ \mathcal{H}_- + \mathrm{gr} +$ miniversal section $+ \dots$

$\approx>$ Frobenius manifold.

Proposition 2.39 $\Xi \in R$.

$$\Psi = \int_\Xi e^{W_0/\hbar}\,\Omega \text{ is a solution to}$$

$$\left(\frac{1}{n+1}\hbar\partial_\hbar\right)^{n+1}\Psi = \hbar^{-(n+1)}\Psi.$$

Lemma 2.41 $\Xi_0, \dots, \Xi_n$ basis $\Rightarrow$ " " fundamental system of solutions.

2.39 Proof $\quad x_0 \cdots x_n = 1$

$$\omega_i = x_0 \cdots x_{i-1}\,\Omega \qquad \Omega_j = \pm\frac{dx_1}{x_1} \wedge \dots \wedge \widehat{\frac{dx_j}{x_j}} \wedge \dots \wedge \frac{dx_n}{x_n}$$

$$= dx_1 \cdots dx_{i-1}\,\frac{dx_i}{x_i} \cdots \frac{dx_n}{x_n}$$

$$(d + \hbar^{-1} dW_0 \wedge)\, x_0 \cdots x_{i-1}\,\Omega_j$$

$$= \left((x_j\partial_j - x_0\partial_0)(x_0 \cdots x_{i-1}) + \hbar^{-1}(x_j - x_0)(x_0 \cdots x_{i-1})\right)\Omega$$

$$\in \mathrm{im}(d + \hbar^{-1} dW_0 \wedge).$$

$$W_0 = x_1 + \dots + x_n + \frac{1}{x_1 \cdots x_n}$$

$$dW_0 = dx_1 + \dots + dx_n + \frac{-1}{x_1^2 \cdots x_n}dx_1 + \cdots$$

$$dW_0 \wedge \Omega_j = x_j\,\Omega - x_1 \cdots x_n\,\Omega$$

$$= (x_j - x_0)\,\Omega$$

$$\Rightarrow \left[(x_j\partial_j + 1)(x_0 \cdots x_{i-1}) + \hbar^{-1}(x_j - x_0)(x_0 \cdots x_{i-1})\right]\Omega$$

$\in$ image.

Sum over all j.

$$\Rightarrow i\omega_i + \hbar^{-1}(W_0 - (n+1)x_i)\,\omega_i \in \text{image}$$

$\Rightarrow \quad \dots$

$$\Rightarrow \frac{1}{(n+1)^{n+1}}\left(-\nabla^{GM}_{\hbar\partial_\hbar}\right)^{n+1}[\Omega] = \hbar^{-(n+1)}[\Omega].$$

$$\text{"}\ Y(-,-) = (\nabla_Y -, -) + (-, \nabla_Y -)\ \text{"}. \qquad \blacksquare$$

$$\sum_j (x_j\partial_j + 1)(x_0 \cdots x_{i-1})\Omega = (i - (n+1) + n+1)\,\omega_i = i\,\omega_i$$

Prop^n 2.40

$$\hbar^{-(n+1)\alpha} = \sum_{i=0}^{n} \frac{(-(n+1))^i \log^i \hbar}{i!}\,\alpha^i \quad \left(\text{in } \mathbb{C}[\alpha]/\alpha^{n+1}\right)$$

Then the coefficients of

$$\xi(\hbar,\alpha) = \hbar^{-(n+1)\alpha} \sum_{d=0}^{\infty} \hbar^{-(n+1)d} \prod_{i=1}^{d} \frac{1}{(\alpha+i)^{n+1}}$$

form a fundamental system of solutions to $(*)$.

Lemma 2.41 $\Xi_0, \dots, \Xi_n$ basis $\Rightarrow$ $\int_{\Xi_i} e^{W_0/\hbar}\,\Omega$ system of fundamental sol^{n}s

Proof Wronskian is nowhere vanishing .. ∎

Choose a basis $\Xi_0, \dots, \Xi_n$ of R such that

$$\sum_{i=0}^{n} \alpha^i \int_{\Xi_i} e^{W_0/\hbar}\,\Omega = \xi(\hbar,\alpha)$$

"dual of Ξ_k"

"the multivaluedness of $\hbar^{-(n+1)\alpha}$ cancels with that of α^k
so $\hbar^{-(n+1)\alpha}\alpha^k$ is single valued".

∴ "we can view" $\hbar^{-(n+1)\alpha}\alpha^k$ as a section of $\mathcal{R}^\vee$. Since α^k is flat ($\in R^\vee$)

$$\nabla\left(\hbar^{-(n+1)\alpha}\alpha^k\right) = 0.$$

So $\hbar^{-(n+1)\alpha}\alpha^k \in \mathcal{H} := \{ s \in \mathcal{E} \otimes \mathcal{O}_M\{\hbar^{\pm 1}\} : \nabla s = 0 \}$

form a basis, as a free $\mathbb{C}\{\hbar^{\pm1}\}$-module.

$$\mathcal{H}_- = \left\langle (\hbar\alpha)^k \hbar^{-1} \hbar^{-(n+1)\alpha} \right\rangle_{\mathcal{O}(\mathbb{P}^1 \setminus 0)}$$

Prop^n 2.43 1) $\mathcal{H} = \mathcal{E}_0 \oplus \mathcal{H}_-$

2) $\mathcal{H}_-$ is isotropic wrt symplectic form

$$\overline{\Omega}(s_1, s_2) = \operatorname{Res}_{\hbar=0}\left(s_1(-\hbar), s_2(\hbar)\right) d$$

3) Gr preserves $\mathcal{H}_- \longleftrightarrow$ log pole @ ∞

$\Omega_0 \in \hbar\mathcal{H}_-$ $\left(\begin{matrix}[\Omega] \\ \text{at } 0 \in M\end{matrix}\right)$ ....

$s_0 = \tau(\Omega_0 \otimes 1)$

is a miniversal section.

Therefore by Thm 2.26 we obtain a Frobenius manifold structure on $(M, 0)$.