Geom Langlands & CFT : 1 : Intro & Number Thy

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Part I – Number Thy ("origin story")

Part II – Geometric Langlands

Part III – Conformal field theory

Intro

Motivation: Langlands conj. appeared in String theory as (mainly) S-dualities of Strings. $\rightsquigarrow$ "formalizing" this kind of Physics would give us some/many examples where GLC holds.

More vague terms: 2-dim reductions of 4D "D-branes" $\rightsquigarrow$ 't Hooft Eigensheaves

Classical LC:

Vaguely: "number theoretic data"

$\Updownarrow \rightsquigarrow$ Fourier transform

"automorphic form data"

We will work towards statements like

Taniyama-Shimura Conjecture

$\hookrightarrow$ Proof of Fermat's Last thm

Get some concrete results for function fields instead of number fields

Moving from arbitrary char. to 0 gives us the GLC.

Bun_n - moduli space of rank-n holomorphic vector bundles on X an $\mathbb{C}$-alg curve

Attach an object called <u>Hecke eigensheaf</u>

$$\boxed{\begin{array}{l}\text{hol rk-}n\text{ vector bdle } E \\ + \text{ hol connection on } X\end{array}} \xrightarrow[\sim]{GLC} \boxed{\begin{array}{l}\text{Hecke eigensheaves} \\ \text{on } Bun_n\end{array}}$$

Here a Hecke eigensheaf is a $\mathcal{D}$-mod on Bun_n satisfying some condition def. by E

This is also expected to work "equivariantly"

- consider a $\mathbb{C}$-reductive gp G with Lang. dual ${}^LG$

In this case:

$$\boxed{\text{holom. } {}^L G\text{-bundles w/ connection on } X} \xrightarrow{\text{GLC}} \boxed{\text{Hecke eigensheaves on } \mathrm{Bun}_G}$$

How to get GLC from CFT:

<u>Rem</u> $\mathcal{E}$ a hol. v.b on a mfd M with <u>flat</u> conn. ∇

$\rightsquigarrow$ <u>holonomic</u> $\mathcal{D}$-mod. on M

"sheaf hol diff. operators on M"

If ∇ is only <u>proj.</u> flat, then

$\rightsquigarrow$ <u>twisted</u> $\mathcal{D}$-mod

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One can use the method:

"conformal blocks construction"

$\rightsquigarrow$ (twisted) $\mathcal{D}$-modules

For "nice" CFTs, e.g. WZW model these $\mathcal{D}$-modules turn out to be proj flat

For other CFTs you can get singularities along divisors $\rightsquigarrow$ Hecke eigensheaves

One of these examples, to be discussed in Part VII, we find families of $\mathcal{D}$-modules depending on

<u>${}^L G$-opers</u>

Beilinson-Drinfield's "mavelous result"

D-mod corr ${}^{L}G$-oper $E \longleftrightarrow$ Hecke eigensheaf with eigenvalue E.