An inverse problem of digital signal processing

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An important problem of digital signal processing is the so-called frequency analysis problem: Determine the different frequencies $\omega_j \in [0, \pi]$, the coefficients α_j, $\beta_j \in \mathbb{R}$, and the parameter $M \in \mathbb{N}$ from the sampled data $f(k)$ $(k = 0, \ldots, 2N)$ of a signal

$$f(x) = \frac{\alpha_0}{2} + \sum_{j=1}^{M} (\alpha_j \cos(\omega_j x) + \beta_j \sin(\omega_j x)) \quad (x \in \mathbb{R}).$$

This is a nonlinear inverse problem which can be simplified by original ideas of G. de Prony (1795).

In the talk, we report on new results of an approximate Prony method. The classical Prony method is numerically unstable such that numerous modifications were attempted to improve its numerical behavior. Our results are based on papers of G. Beylkin and L. Monzón (Appl. Comput. Harmon. Anal. **19** (2005), 17-48). The nonlinear problem of finding the frequencies and coefficients can be split into two problems. To obtain the frequencies, we solve an eigenvalue problem of the Hankel matrix $\mathbf{H} = (f(k+l))_{k,l=0}^{N}$ and find the frequencies via roots of an eigenpolynomial. To obtain the coefficients, we use the frequencies to solve a linear Vandermonde–type system. In contrast to G. Beylkin and L. Monzón, we apply matrix perturbation theory such that we can describe the numerical behavior of the approximate Prony method in detail. Numerical experiments show the performance of our method.

The talk is based on joint work with Daniel Potts.