Application of Spindle Speed Variation for Chatter Suppression in Turning

Andreas Otto\textsuperscript{a},*, Günter Radons\textsuperscript{a}

\textsuperscript{a}Institute of Physics, Chemnitz University of Technology, 09107 Chemnitz, Germany

Abstract

In the present paper the chatter instability of variable speed machining is studied. Though, there exist numerical methods for the computation of the stability lobes for variable speed machining, especially in turning processes the potential of an active spindle speed variation for chatter suppression is mostly unexploited. In the case of a slowly time-varying spindle speed, which is practicable on a real machine tool, the stability behavior with a time-varying spindle speed is connected to the stability behavior with constant spindle speeds. This so-called frozen time approximation helps to understand the stabilizing mechanism of turning with spindle speed variation. Strategies for tuning the parameters of the speed variation for an optimal stabilization are developed. The results presented here are useful for a practical implementation of variable speed machining to increase the productivity without any negative effect due to the variation of the spindle speed.

Keywords:
spindle speed variation, chatter, time-varying delay, turning

1. Introduction

Machine tool chatter is characterized by large vibrations in machining, which cause poor surface finish, reduce the lifetime of machine tool components and increase tool wear. The feedback between the outer modulation of the chip to the current cutting force, and consequently, the inner modulation of the chip is called regenerative effect. Hence, the productivity of stable machining without disadvantageous vibrations depends on the time delay as the time between two subsequent cuts and is limited by the so-called stability lobes of the regenerative effect (Tobias, 1961; Danek et al., 1962). A continuous spindle speed variation
(SSV) can create a time-varying delay. As a result, the suppression of unfavorable phase lags between inner and outer chip modulation can reduce chatter vibrations (Stöferle and Grab, 1972). The underlying mathematical model of turning with SSV are delay differential equations (DDEs) with time-varying delay. The stability of such equations can be calculated numerically, for example, by the semidiscretization method (Insperger and Stépán, 2004), time domain simulations or in the frequency domain by a multifrequency approach (Jayaram et al., 2000; Zatarain et al., 2008). Michiels et al. (2005) present an analytical method for the stability analysis of DDE with fast time-varying delay and Al-Regib et al. (2003) proposed a method for the optimal selection of SSV parameters, which holds under the same assumptions. However, in a practical implementation of a SSV the acceleration of the spindle is limited by the spindle drive power and only slow variations of the delay are possible. In this case the assumptions of (Al-Regib et al., 2003; Michiels et al., 2005) are not fulfilled.

In contrast to hardware modifications of the production system a SSV can be simply implemented in the CNC unit of any machine tool. Moreover, contrary to spindle speed selection strategies (Bediaga et al., 2009), the SSV increases the lowest stability border in a wide range of nominal spindle speeds and can improve productivity by a suppression of chatter vibrations in a general way. In this way SSV is a promising method for the suppression of chatter vibrations in thin wall part turning (Lorong et al., 2011), where the structural behavior of the workpiece changes during the process. Furthermore, it was successfully applied for stabilizing turning (Yilmaz et al., 2002), milling (Zatarain et al., 2008; Otto et al., 2011) and grinding (Barrenetxea et al., 2009) processes. Nevertheless, the application of SSV is not well accepted in industry. On the one hand, there are objections on the negative effects of variable speed machining such as possible destabilizations, temporal chatter vibrations and ongoing de- and acceleration of the spindle. On the other hand, it is difficult to find optimal parameters of the SSV and to estimate the effectiveness of the stabilization with respect to constant speed machining. Hence, the potential of a general strategy for the stabilization of the cutting process by a simple electronic way is not being exploited.

In this paper a simplified theory for the stability analysis of turning with slowly time-varying spindle speed is presented. On the basis of this so-called adiabatic or frozen time approach, it is possible to understand the stabilizing mechanism and to estimate the efficiency of an application of a SSV. Based on the results of the theoretical study, strategies for an optimal selection of the parameters of the SSV are developed. Furthermore, it is shown, that the negative effects of an implementation of a SSV can be kept to a minimum. The paper is organized as follows. In Sec. 2 the model for turning processes with SSV is described. The theory for the stability analysis of DDE with a slowly time-varying delay is presented in Sec. 3. Thereafter, the stability behavior for turning with SSV is systematically investigated on the basis of the frozen time approach and guidelines for a practical implementation of a SSV for chatter suppression are given.
2. Model

2.1. Structural model

The response of the structure of a machine tool to the cutting force can be modeled by a system of harmonic oscillators specifying $N$ normal modes of the structure. The cutting force of strength $F$ acts on each mode $i$ with modal mass $m_i$, relative damping factor $\zeta_i$ and natural angular frequency $\omega_{ni}$.

$$m_i \left( \ddot{q}_i(t) + 2\zeta_i\omega_{ni}\dot{q}_i(t) + \omega_{ni}^2 q_i(t) \right) = F \cos \alpha_i$$

The angle $\alpha_i$ is the angle between the direction $\hat{e}_F$ of the cutting force and the $i$th eigenmode of the structure (see Fig. 1).

![Figure 1: Turning process with mode $\hat{e}_{q,i}$, cutting force $\hat{e}_F$ and feed $\hat{e}_x$ direction.](image)

2.2. Cutting force

The dependence of the modulus $F$ of the cutting force on the depth of cut $b$ and the chip thickness $h$ is assumed to be linear (Altintas, 2000).

$$F = K_c bh$$

The cutting force coefficient $K_c$ and the angle $\beta$ between the cutting force direction $\hat{e}_F$ and its tangential component (cf. Fig. 1) is preferably specified experimentally for a special tool-workpiece combination. It can vary with the feed per round $h_0$ and the cutting speed. It is also possible to consider a more precise cutting force model, but the influence of the cutting force model on the stability of the cutting process with SSV is very small.

The value of the uncut chip thickness $h$ is composed of the feed per round $h_0$ and dynamical displacements into the feed direction $\hat{e}_x$ at the inner $x(t)$ and the outer $x(t-\tau(t))$ surface of the chip.

$$h(t) = h_0 + x(t) - x(t-\tau(t))$$

The dynamical displacements $x$ can be expressed in terms of the modal displacements $q_j$ as

$$h(t) = h_0 + \sum_{j=1}^{N} \sin(\alpha_j + \beta)(q_j(t-\tau(t)) - q_j(t)).$$
The static part of the cutting force due to the feed per round $h_0$ has no influence on the dynamical stability of the system and can be ignored. The dynamic part of the cutting force depends linearly on the instantaneous $q_j(t)$ and the retarded displacements $q_j(t - \tau(t))$ of all $N$ modes.

2.3. Equations of motion

Putting all modal displacements in a vector $\mathbf{q} = (q_1, \ldots, q_N)^T$, the dynamical system for mechanical vibrations at the tool tip can be written as

$$M\ddot{\mathbf{q}}(t) + C\dot{\mathbf{q}}(t) + K\mathbf{q}(t) = K_c b D [\mathbf{q}(t - \tau(t)) - \mathbf{q}(t)]. \quad (5)$$

The matrices $M$, $C$ and $K$ are diagonal matrices containing the masses $m_i$, the damping coefficients $2\zeta_i\omega_n m_i$ and stiffnesses $m_i\omega_n^2$ for each mode $i$ on their main diagonal. The elements $d_{ij}$ of the matrix $D$ are called directional factors (cf. Zatarain et al., 2010) and can be specified with equations (1), (2) and (4) by

$$d_{ij} = \sin(\alpha_j + \beta) \cos \alpha_i. \quad (6)$$

If the limiting depth of cut $b$ is dominated only by one flexible mode of the structure, system (5) is similar to the well-known one degree of freedom turning model, which is often studied in the literature (Al-Regib et al., 2003; Insperger and Stépán, 2004; Michiels et al., 2005; Yilmaz et al., 2002).

2.4. Variable spindle speed and variable delay

The calculation of the uncut chip thickness $h(t)$ requires knowledge of the dynamical displacements $q(t - \tau(t))$ of the tool at the previous cut and the same angular position of the workpiece. The time delay $\tau(t)$ equals the time for one revolution of the workpiece. For a time-varying spindle speed $\Omega(t)$ in rpm the angular position of the workpiece at time $t$ in rad is

$$\phi(t) = 2\pi\int_0^t \frac{\Omega(t')}{60} dt'. \quad (7)$$

The time delay $\tau(t)$ can be written in implicit form as

$$2\pi\int_{t-\tau(t)}^t \frac{\Omega(t')}{60} dt' = \phi(t) - \phi(t - \tau(t)) = 2\pi. \quad (8)$$

Assuming the usual case of a strictly increasing function $\phi(t)$, its inverse $\phi^{-1}$ is unique and the exact form of the time-varying delay $\tau(t)$ can be expressed explicitly by

$$\tau(t) = t - \phi^{-1}(\phi(t) - 2\pi). \quad (9)$$

Contrary to the common expectation, a time-varying spindle speed does not always result in a time-varying delay. For a periodic SSV with period $T$ and nominal spindle speed $\Omega_0$ the following periodicity condition holds

$$\phi(t + T) = \phi(T) + 2\pi \frac{\Omega_0}{60} T. \quad (10)$$
For the special case of $T = \frac{60}{\Omega_0}$ the time delay $\tau(t) = T$ is constant, independent of the form of the SSV $\Omega(t)$. This is due to the fact, that after each rotation of the spindle, the spindle speed is the same and the same time has passed. It can be also verified by putting the periodicity condition of Eq. (10) into Eq. (9).

In general the relative amplitude of the delay variation $R_\tau$ is different from the relative amplitude $R_\Omega$ of the spindle speed variation. It can be defined as

$$R_\tau = \frac{\max(\tau(t)) - \min(\tau(t))}{2} \frac{\Omega_0}{60}. \quad (11)$$

The most common SSV is a sinusoidal time-varying spindle speed

$$\Omega(t) = \Omega_0(1 + R_\Omega \sin(2\pi \nu_m t)), \quad (12)$$

with the relative amplitude $R_\Omega$ and the frequency $\nu_m$. In Fig. 2 the amplitude ratio $\rho_A = \frac{R_\tau}{R_\Omega}$ of a sinusoidal SSV, calculated with the exact relation of Eq. (9) for the time-varying delay $\tau(t)$, is shown for three different values of $R_\Omega$ as function of relative modulation frequencies

$$R_F = \frac{60}{\Omega_0} \nu_m. \quad (13)$$

Only for small relative frequencies $R_F$ and amplitudes $R_\Omega$ of the SSV, the amplitude ratio $\rho_A$ is approximately one. In fact, for small relative frequencies $R_F$ the variation of the spindle speed in the integral of Eq. (8) can be neglected and the time delay can be approximated by

$$\tau(t) \approx \frac{60}{\Omega(t)}. \quad (14)$$
In this case the amplitude ratio $\rho_A$ is larger than one and increases with increasing $R_\Omega$ (see Fig. 2). For integer values $R_F$ the time delay is constant, which corresponds to the case discussed above. Hence, a small relative frequency $R_F$ and a large relative amplitude $R_\Omega$ of the SSV are suited to create a variable time delay with a large relative amplitude $R_\tau$.

It can be useful to obtain a constant absolute amplitude $A_m$ of the delay modulation as function of the nominal spindle speed $\Omega_0$. The absolute amplitude $A_m$ of the delay variation can be determined by putting the maximum and minimum value of the SSV of Eq. (12) into Eq. (14):

$$A_m(\Omega_0) = \frac{60R_\Omega}{\Omega_0(1 - R_\Omega^2)}.$$

Neglecting the term $R_\Omega^2$, a roughly constant absolute amplitude $A_m$ of the delay variation as function of the nominal spindle speed $\Omega_0$ can be obtained by

$$R_\Omega(\Omega_0) \approx A_m \frac{\Omega_0}{60}.$$

This means, that for higher nominal spindle speeds $\Omega_0$ higher relative amplitudes $R_\Omega$ are necessary to get comparable absolute amplitudes $A_m$ of the delay modulation. Furthermore, the higher the nominal spindle speeds $\Omega_0$, the higher are the motor torques for the implementation of the SSV. These are two reasons why the application of SSV is more beneficial for lower nominal spindle speeds.

The systematical stability analysis of turning processes with SSV in Sec. 4 is done for a sinusoidally time-varying delay

$$\tau(t) = \tau_0 + A_m \sin(2\pi\nu_m t)$$

with a fixed absolute amplitude $A_m$ around the nominal delay $\tau_0$.

3. Theory

3.1. Eigenmode expansion of DDE with constant delay

From a mathematical point of view the dynamical model for turning with SSV in Eq. (5) is a DDE with time-varying delay. At first the eigenmode expansion for DDEs with constant delay is presented following Amann et al. (2007). If the spindle speed $\Omega$ is constant, the time delay is also constant $\tau = \frac{60}{\Omega}$ and Eq. (5) can be rewritten in first-order form

$$\dot{y}(t) = Ay(t) + By(t - \tau).$$

The solution of Eq. (18) can be expressed in terms of a linear combination

$$y(t) = \sum_{l=0}^{\infty} c_l u_l(t - t^0).$$

of infinitely many eigenmodes

\[ u_i(\theta) = \exp(s_i \theta) \hat{u}_i, \]

with an initial time \( t^0 \). Each eigenmode \( u_i(\theta) \) is a solution of Eq. (18). The values \( s_i \) and \( \hat{u}_i \) are specified by the characteristic equation, which can be obtained by putting \( y = u_i \) into Eq. (18)

\[ (Is_I - A - B \exp(-s_l \tau)) \hat{u}_l = S_l \hat{u}_l = 0, \]

where \( I \) is the identity matrix. The exponents \( s_l \) are the infinitely many roots of the transcendental equation \( \det S_l = 0 \) and the \( \hat{u}_l \) are the corresponding eigenvectors to the zero eigenvalue of \( S_l \). Both \( s_l \) and \( \hat{u}_l \) depend on the coefficient matrices \( A, B \) and on the time delay \( \tau \). The coefficient \( c_l \) of the \( l \)th eigenmode in Eq. (19) depends on the initial function \( y(\theta) = \phi(\theta), t^0 - \tau \leq \theta \leq t^0 \)

\[ c_l = \int_{t^0-\tau}^{t^0} v^*_l(\theta - t^0) \phi(\theta) d\theta, \]

where \( v^*_l(\theta) \) is the eigenfunction dual to \( u_l(\theta) \) (Amann et al., 2007), i.e.

\[ \int_{t^0-\tau}^{t^0} v^*_l(\theta - t^0) u_m(\theta - t^0) d\theta = \delta_{lm}, \]

where \( \delta_{lm} \) denotes the Kronecker delta.

By arranging the eigenmodes \( u_l(\theta) \) according to the real part \( \lambda_l \) of the associated characteristic roots \( s_l \) with \( s_0 \) being the root with the largest real part, \( u_0(\theta) \) is the most unstable eigenmode of the solution of Eq. (18). If \( s_0 \) is split into its real and imaginary part

\[ s_0 = \lambda_0 + i\omega_0, \]

the DDE is stable if \( \lambda_0 < 0 \) and unstable with the chatter frequency \( \omega_0 \) if \( \lambda_0 > 0 \). Furthermore, after a sufficiently large time \( t \) the solution \( y(t) \) of (18) is dominated only by the most unstable eigenmode \( u_0(\theta) \).

### 3.2. Frozen time approach for DDE with slowly time-varying delay

Due to technical limitations, the application of a SSV often results in a slowly varying periodic time delay. If the period \( T \) of the delay variation is divided into \( P \) time intervals \( I^k = [t^k, t^{k+1}] \) with uniform interval length \( t^{k+1} - t^k = \Delta t \) and \( P\Delta t = T \), the time-varying delay can be assumed to be frozen in the intervals \( I^k \), i.e.

\[ \tau(t) \approx \tau(t^k) = \tau^k, \quad \text{for} \ t \in I^k. \]

Now the intervals \( I^k \) are studied separately and the \( s_l^k, \hat{u}_l^k, u_l^k(\theta), v_l^k(\theta) \) and \( c_l^k \) are the exponents, eigenvectors, eigenmodes, dual eigenfunctions and coefficients of the eigenmode expansion of Eq. (18) in the time interval \( I^k \) with the delay \( \tau = \tau^k \).
The asymptotic behavior of the solution of the DDE (18) is dominated by the most unstable eigenmode \( u_0(\theta) \). For the frozen time approximation with a slowly time-varying delay an initial function \( \phi(\theta) \) with \( \theta \in [\tau^0 - \tau^0, t^0] \) is chosen in such a way, that the coefficients \( c_l^0 \) of the eigenmode expansion in the first interval \( I^0 \) are

\[
\begin{align*}
c_l^0 &= \begin{cases} 
1, & l = 0 \\
0, & \text{otherwise}. 
\end{cases}
\end{align*}
\]  

(26)

Then, the solution \( y = y^0 \) in \( I^0 \) can be characterized solely by

\[
y^0(t) = u_0^0(t - t^0), \quad \text{for } t \in I^0.
\]  

(27)

The frozen eigenmode \( u_0^0 \) is specified by the most unstable characteristic root \( s_0^0 \) of Eq. (21) for the frozen delay \( \tau = \tau^0 \).

The coefficients \( c_l^1 \) of the eigenmode expansion in the next interval with \( t \in I^1 \) and frozen delay \( \tau = \tau^1 \) are determined according to Eq. (22) by

\[
c_k^1 = \int_{t_{k-1}}^{t_k} u_1^0(\theta - t_k)y^{k-1}(\theta)d\theta,
\]  

(28)

with large enough \( \Delta t \) and \( k = 1 \). In the limit of a slowly time-varying delay the modifications of the most unstable eigenmode due to a variation of the delay from \( \tau^0 \) to \( \tau^1 \) can be neglected, \( u_0^0 \approx u_1^0 \), and with Eq. (27) and the orthogonality condition in Eq. (23) the coefficients in the interval \( I^1 \) are approximately

\[
c_l^1 = \begin{cases} 
\exp(s_0^0 \Delta t), & l = 0 \\
0, & \text{otherwise}. 
\end{cases}
\]  

(29)

The solution \( y^1 \) in the interval \( I^1 \) is again determined solely by the most unstable frozen eigenmode \( u_1^0 \) of the system with constant frozen delay \( \tau^1 \)

\[
y^1(t) = \exp(s_0^0 \Delta t)u_1^0(t - t^1).
\]  

(30)

After one period of the delay variation at time \( t = t^0 + T = t^P \) the ongoing frozen time approximation of the solution leads to

\[
y^P(t^P) = \hat{u}_0^P \prod_{k=0}^{P-1} c_k^0 = \hat{u}_0^P \exp \left( \Delta t \sum_{k=0}^{P-1} s_k^0 \right).
\]  

(31)

Since \( \tau^P = \tau^0 \), the eigenvector \( \hat{u}_0^P \) is equal to the solution \( y^0(t^0) = \hat{u}_0^0 \) one period ago and the largest Floquet multiplier \( \mu_0 \) of the periodic DDE can be approximated in the limit of a slowly time-varying delay by

\[
\mu_0 = \exp(T \bar{s}_0), \quad \text{with } \bar{s}_0 = \frac{1}{P} \sum_{k=0}^{P-1} s_k^0.
\]  

(32)

Hence, the real part \( \lambda_0^0 \) of the largest Floquet exponent \( \bar{s}_0 \) is the average of the real part \( \lambda_0^k \) of the largest exponents of the subsystems with frozen delays \( \tau^k \).
The system is

\[
\begin{align*}
\text{stable, for} & \quad \tilde{\lambda}_0 < 0 \\
\text{marginal stability, for} & \quad \tilde{\lambda}_0 = 0 \\
\text{unstable, for} & \quad \tilde{\lambda}_0 > 0.
\end{align*}
\]

For the computation of the stability lobes of the frozen time approach, the calculation of the characteristic roots \(s_k^0\) with the largest real part is necessary for the DDE with all adopted constant delays \(\tau^k\). This can be done for example with the semidiscretization method (Insperger and Stépán, 2002). Similar to the statement of Michiels et al. (2005) for the stability approximation in the limit of a fast time-varying delay, numerical simulations are the best way to check the validity of the frozen time approach. In this way the frozen time approach is not necessarily an effective and reliable way to calculate the stability lobes of machining with SSV. But indeed the presented theory helps to understand the mechanism of chatter suppression due to an application of a SSV and to find strategies for an optimal selection of the SSV parameters. For a milling process with SSV the approximation of the frozen time approach agree very well with stability results of numerical simulations and experiments (Otto et al., 2011).

3.3. Asymptotic and temporal stability

As mentioned by Seguy et al. (2010) asymptotically stable machining with SSV can be practically unstable and vice versa. Asymptotic stability specifies the exponential behavior of the solution over the principle period \(T\) of the delay modulation. Temporal stability is related to the temporal evolution of the vibration amplitude during one period of the delay modulation \(T\). For a slowly time-varying delay the asymptotic stability is defined by Eq. (33). Considering only the real part \(\lambda_k^0\) of the frozen time exponents the chatter amplitude \(\gamma^m\) in the interval \(I^m\) can be approximated in analogy to Eq. (31) by

\[
\gamma^m(t^m) = \tilde{u}^m_0 \exp \left( \Delta t \sum_{k=0}^{m-1} \lambda_k^0 \right).
\]

Especially for a slow SSV the frozen delays \(\tau^k\) can be located for a relatively long time in an unstable cutting condition with \(\lambda_k^0 > 0\). In these regions the amplitude of the chatter vibration \(\gamma^m\) increases, which is referred to as a temporal instability. Nevertheless, the delay \(\tau^k\) can reenter stable cutting conditions with \(\lambda_k^0 < 0\) for which the amplitude \(\gamma^k\) decreases. The asymptotic stability behavior of the system is completely independent of the temporal stability behavior. There can be large temporal instabilities, though the system over the complete period \(T\) is stable with \(\lambda_0 < 0\).

A reduction of the temporal vibration amplitude \(\gamma^m\) can be achieved by increasing the period \(T\) of the delay variation. In this case one period of the time delay can be discretized by the same number \(P\) of \(\tau^k\) with the same exponents \(\lambda_k^0\) but a smaller step size \(\Delta t\), which leads to a smaller temporal vibration amplitude \(\gamma^m\).
4. Application of SSV in turning

With the presented theoretical results on the stability of DDE with time-varying delay it is possible to identify guidelines for a successful and efficient implementation of a SSV in turning.

4.1. Form of the SSV

From a theoretical point of view the form of the SSV has a minor influence on the chatter instability. A modification of the form of the time-varying delay results in a modified weighted sum of the $\lambda_k^0$ in Eq. (32) and a slightly different stability behavior. However, an optimization of the form of the SSV is only useful for a well-specified spindle speed and is mostly no more than fine tuning of the stability lobes. In this paper, harmonical variations of the spindle speed are studied, since they are advantageous for practical implementations. Remarks on the robustness of the SSV, if the real trajectory of the spindle speed profile does not match exactly with the theoretically desired one, are made in Sec. 5.3.

4.2. Effect of the eigenfrequency

The effect of the eigenfrequencies $\omega_n$ of the machine-tool structure on the effectiveness of a SSV for chatter suppression is exemplarily illustrated for two one degree of freedom systems ($N = 1$) with exactly the same stiffness $m \omega_n^2 = 0.8N/\mu m$, damping ratio $\zeta = 0.03$, cutting force coefficient $K_c = 800N/mm^2$, mode angle $\alpha = 89.25^\circ$ and cutting force angle $\beta = 15^\circ$ but different natural frequencies $\omega_n$. The stability lobes for both systems were calculated for constant delay $A_m = 0$ (thick lines) and for a sinusoidally varying delay with $A_m = 0.004s$ and $\nu_m = 1Hz$ (thin lines). The results for nominal delays $0.01s \leq \tau_0 \leq 0.03s$, corresponding to nominal spindle speeds $\Omega_0$ between 2000rpm and 6000rpm, are shown in Fig. 3 and compared to the results of the frozen time approach (dashed lines). Stable cutting is possible for chip widths below each particular stability lobe.

Different structural eigenfrequencies but same stiffnesses and damping ratios does not affect the minimum of the stability lobes for constant spindle speeds. However, in Fig. 3 a) with $\omega_n = 2\pi 100s^{-1}$ the stability lobes are four times more extended in parameter $\tau_0$ than the lobes in Fig. 3 b) with a four times larger eigenfrequency $\omega_n = 2\pi 400s^{-1}$. This has an influence on the stability of the process with time-varying delay. In Fig. 3 a) for $\omega_n = 2\pi 100s^{-1}$ the stability lobes with variable delay fit, especially near its minima, into the results of the frozen time approach. In the peak points between two lobes (e.g. near cutting condition 'A') the lobes with variable delay are much higher than the frozen time approximation. In contrast to the prediction of the frozen time approach, there is nearly no destabilization of the system due to a time-varying delay.

In Fig. 3 b) for $\omega_n = 2\pi 400s^{-1}$ the lobes with SSV are always much higher than the frozen time approximation and the results for constant delay. This can be explained as follows. For high eigenfrequencies $\omega_n$ with small stability lobes the characteristic roots $\xi_k$ and the shape of the most unstable frozen eigenmodes $u_k^0$ are much more sensitive to a change of the time delay $\tau_k$. The solution in
the frozen intervals $I^k$ can be no longer described only by the evolution of the most unstable eigenmode $u^k_0$. The structural vibrations are stabilized by further eigenmodes $u^k_l$ with $\lambda^k_l < \lambda^k_0, l > 0$ and the frozen time approach underestimates the stability regions.

In other words, higher eigenfrequencies $\omega_n$ lead to higher chatter frequencies $\omega_0$, a larger variation of the phase lag $\omega_0 \tau(t)$ between inner and outer chip modulation and a stronger dissipation of the most unstable frozen eigenmode. A very strong and efficient stabilization of the turning process by an implementation of a SSV is possible, which was already shown in experiments by Yilmaz et al. (2002).

4.3. Effect of the structural damping

The stability lobes for the same parameters as in Fig. 3 but with a lower damping ratio $\zeta = 0.006$ are shown in Fig. 4. Again the results for constant delay (thick), sinusoidally varying delay (thin) and the corresponding frozen time approximation is presented (dashed) for the two values of the structural eigenfrequency $\omega_n$. The local minima of the stability lobes for constant delay for the weakly damped structure with $\zeta = 0.006$ (Fig. 4) are roughly five times lower than the minima of the lobes for the structure with $\zeta = 0.03$ (Fig. 3). Nevertheless, the maximum stable chip widths in the peak points between two lobes are in the same region for both damping ratios. This stability variation for constant delay affects also the stability behavior of the system with variable time delay.

In Fig. 4 a) for $\omega_n = 2\pi 100s^{-1}$ there is a significant destabilization of the turning process due to the SSV near the peak points between two stability lobes.
of the system with constant delay (e.g. near cutting condition 'B'). In Fig. 4 b) for \( \omega_n = 2\pi 400 \text{s}^{-1} \) the stability lobes with SSV are still higher than the frozen time approximation but not always higher than the stability results for constant delay in contrast to the behavior for \( \zeta = 0.03 \) (cf. Fig. 3 b)). In general, for turning processes the stability prediction of the frozen time approximation is more accurate for weaker damping ratios of the machine-tool structure (cf. Figs. 3 and 4).

The distinct stability behavior of turning with time-varying delay for different damping ratios is studied in more detail by comparing the dynamics at cutting condition 'A' with \( \zeta = 0.03, \tau_0 = 0.02 \text{s} \) and \( b = 24 \text{mm} \) (see Fig. 3) and the dynamics at cutting condition 'B' with \( \zeta = 0.006, \tau_0 = 0.02 \text{s} \) and \( b = 8 \text{mm} \) (see Fig. 4). Both points 'A' and 'B' are stable cutting conditions near a peak point of two lobes for constant delay. In Fig. 5 a) and b) the time-varying delay \( \tau(t) \) and in Fig. 5 i) and j) the structural vibrations \( q(t) \) for cutting conditions 'A' and 'B' are shown as function of time for one period of the delay variation. The imaginary part \( \omega_{kl}^k \) and the real part \( \lambda_{kl}^k \) of the two \((l = 0, 1)\) most unstable characteristic roots \( s_{kl}^k \) of Eq. (21) dependent on the frozen delays \( \tau_k \) at the time steps \( t^k \) are shown in Fig. 5 c), d) and e), f), respectively. Fig. 5 g) and h) shows the coefficients \( c_{kl}^k \) of the two \((l = 0, 1)\) most unstable eigenmodes \( u_{kl}^k \), calculated by Eq. (22) with the exact numerical solution \( q(t) \).

At cutting condition 'A' the system with constant delay is unstable with \( \lambda_{00}^k > 0 \) for almost all adopted \( \tau^k \) of the time-varying delay \( \tau(t) \) (see Fig. 5 e) and Fig. 3 a)). However, the system with time-varying delay is stable over one period. After the switching points of the chatter frequencies \( \omega_{00}^k \) (see Fig. 5 c)) of the most unstable eigenmode at \( 4.5s < t < 4.75s \) and \( 5s < t < 5.25s \) the solution

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**Figure 4**: Stability lobes for turning with and without SSV for a weak damping ratio \( \zeta = 0.006 \) and two different eigenfrequencies \( \omega_n \).
q(t) is dominated by the second eigenmode with $c_k^1 >> c_k^0$ (see Fig. 5 g)) which is stable with $\lambda_k^1 < 0$ (see Fig. 5 e)). The vibration amplitude decreases until the solution is again dominated $c_k^0 > c_k^1$ by the most unstable eigenmode (see Fig. 5 g)) for $4.25s < t < 4.5s$ and $4.75s < t < 5s$, where the vibration amplitude increases. Due to the large influence of the second eigenmode ($c_k^1 >> 0$) on the solution $q(t)$ of the DDE, the stability behavior of the system with time-varying delay at cutting condition 'A' cannot be accurately approximated by the frozen time approach, for which the coefficient $c_k^1$ is assumed to be equal to zero. Contrarily, at cutting condition 'B' the influence of the second mode to the solution is much smaller (see Fig. 5 h)). The vibrations $q(t)$ in Fig. 5 j) are dominated by the most unstable mode $u_k^0$ and the predictions of the frozen-time approach are close to the exact stability behavior of the system with time-varying delay.
4.4. Optimal parameters for SSV

It is difficult to find a universal strategy for the selection of optimal amplitudes and frequencies of the SSV. The stabilization depends sensitively on the structural behavior and on the nominal spindle speed as was discussed in Sec. 4.2 and Sec. 4.3, and the maximal acceleration of the spindle is limited. Nevertheless, it is possible to give some guidelines for an optimal selection of the parameters of the SSV.

1. The acceleration of the spindle must be large enough to pass temporarily unstable cutting conditions fast enough. As can be seen from Fig. 5 i) and j), in spite of asymptotically stable cutting conditions the vibration amplitude can become temporarily large. The temporal evolution of the chatter amplitude \( \gamma_m(t) \), defined in Eq. (34), can be used to control the temporal stability behavior.

2. A large amplitude \( A_m \) of the delay variation is preferred to a large frequency \( \nu_m \) of the delay variation. The larger the frequency \( \nu_m \), the smaller is the amplitude ratio \( \rho_A \) of the relative amplitudes of the delay \( R_\tau \) and the spindle speed \( R_\Omega \) (cf. Fig. 2). Furthermore, for larger amplitudes \( A_m \), more switching points of the chatter frequencies \( \omega_0 \) of the most unstable eigenmode are passed through one period of the SSV. The passing of a switching point, which appears for delay values between two stability lobes for constant delay, leads to a stabilization of the solution due to a transfer of vibration energy into more stable eigenmodes \( u_k^l \) with \( l > 0 \) and \( \lambda_k^l < \lambda_0^k \) (cf. Sec. 4.3).

3. The selection of the relative amplitude \( R_\Omega \) of the SSV dependent on the nominal spindle speed \( \Omega_0 \) with a fixed absolute amplitude \( A_m \) of the delay variation according to Eq. (16) is advisable for minimizing the load on the spindle drive system. Especially for small nominal spindle speeds \( \Omega_0 \) and high chatter frequencies \( \omega_0 \) a small relative amplitude \( R_\Omega \) and frequency \( \nu_m \) of the SSV is sufficient for a strong stabilization of the cutting process (cf. Fig. 3 b)).

5. Example

In this section a numerical example for turning is chosen to illustrate the efficiency and robustness of a SSV for application. The structure of the machine-tool is modeled by two uncoupled harmonic oscillators. The parameters and the orientation of the two oscillators are chosen as shown in Tab. 1. The process parameters are \( K_c = 1000 \text{N/mm}^2 \) and \( \beta = 20^\circ \). The relative amplitude \( R_\Omega \) of the sinusoidal SSV is selected dependent on the nominal spindle speed similar to Eq. (16) for achieving a fixed absolute delay amplitude \( A_m \). The exact relation between spindle speed \( \Omega(t) \) and time delay \( \tau(t) \) of Eq. (9) was used for the numerical stability analysis.
Table 1: Structural data of the numerical example

<table>
<thead>
<tr>
<th>Mode $i$</th>
<th>$\alpha_i$</th>
<th>$m_i$</th>
<th>$\omega_0/(2\pi)$</th>
<th>$\zeta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 1$</td>
<td>$80^\circ$</td>
<td>$30,kg$</td>
<td>$120,Hz$</td>
<td>$0.03$</td>
</tr>
<tr>
<td>$i = 2$</td>
<td>$130^\circ$</td>
<td>$0.6,kg$</td>
<td>$1100,Hz$</td>
<td>$0.035$</td>
</tr>
</tbody>
</table>

5.1. Stabilization

In Fig. 6 the stability lobes of the turning process are illustrated for constant speed machining (thick), for a slow SSV with $A_m = 0.001\,s$ and $\nu_m = 0.5\,Hz$ (thin) together with the associated frozen-time approximation (dashed) and for a fast SSV with $A_m = 0.002\,s$ and $\nu_m = 1\,Hz$ (points). For both parameter sets of the SSV the cutting process with SSV is always more stable than for constant spindle speeds. For the slow and the fast SSV up to 20% respectively 60% higher values of stable chip widths $b$ are possible compared to constant speed machining. As expected, the small stability lobes (e.g. $2200\,rpm < \Omega_0 < 2600\,rpm$), which correspond to high chatter frequencies $\omega_0 \approx 2\pi 1100\,s^{-1}$, are more accessible to a stabilization via SSV than the large lobes (e.g. $2600\,rpm < \Omega_0 < 2900\,rpm$), which correspond to low chatter frequencies $\omega_0 \approx 2\pi 120\,s^{-1}$. The lowest stabilization occur for the slow SSV with $A_m = 0.001\,s$ and $\nu_m = 0.5\,Hz$ in the region of the large lobes. In this case, the limiting chip width $b$ with SSV can be accurately approximated by the frozen time approach. In this example, the results of the frozen time approach are a lower bound on the exact stability lobes with SSV.

5.2. Minimization of negative effects of the SSV

The relative amplitudes $R_\Omega$ and the maximum spindle power for an implementation of the SSV depend on the nominal spindle speed $\Omega_0$ as in Eq. (16).
For the slow SSV with $A_m = 0.001s$ the relative amplitudes $R_\Omega$ of the SSV are between 1.7% for $\Omega_0 = 1000rpm$ and 5% for $\Omega_0 = 3000rpm$ and for the fast SSV with $A_m = 0.002s$ the relative amplitudes $R_\Omega$ are between 3.3% and 10%. With an exemplary moment of inertia of $J = 0.1kgm^2$ the maximum power $P_{1000}$, required for the acceleration of the spindle, is between $P_{1000} = 57W$ and $P_{3000} = 1552W$ for the slow SSV and between $P_{1000} = 230W$ and $P_{3000} = 6229W$ for the fast SSV, respectively. However, the maximum power for the SSV is only required to accelerate the spindle, whereas the deceleration of the spindle is done by the cutting torque. On the average over one period no extra energy is consumed due to the implementation of a SSV in comparison to a constant spindle speed.

The relatively small widths of the stability lobes prevent a temporarily excessive increase of the vibration amplitude. Although, the modulation frequency $\nu_m = 0.5Hz$ of the slow SSV is low, numerical simulations have confirmed that no problems with temporal instabilities arise in the above presented example for turning with SSV.

5.3. Robustness

It is possible, that the intended sinusoidal time-varying spindle speed according to Eq. (12) may not exactly be obtained on a real machine during the cutting process. Numerical simulations have shown, that as long as the selected amplitude of the SSV is reached, only small horizontal or vertical displacements of the lobes arise in the stability chart. In order to illustrate this in the present example, the stability lobes of the system with a more general SSV are investigated

$$\Omega(t) = \Omega_0(1 + R_\Omega \sin(2\pi \nu_m t - \delta \sin(2\pi \nu_m t))). \quad (35)$$

The additional term with the amplitude $\delta$ enables the introduction of a periodical phase shift into the pure sinusoidal signal. In Fig. 7 the stability lobes of the example are shown for the fast SSV with $A_m = 0.002s$, $\nu_m = 1Hz$ and three values of $\delta$. The stability lobes for $\delta = 0$ refer to results of the intended SSV already presented in Fig. 6. For $\delta = 0.3$ and $\delta = 0.6$ the acceleration of the spindle is slower than the deceleration. The resulting variance of the time-varying delay is smaller and the limiting depth of cut for the non-ideal sinusoidal spindle speed profile is slightly smaller compared to the results for $\delta = 0$. However, the deviations due to a non-ideal spindle speed profile are small compared to the difference of the stability lobes between constant and variable speed machining.

6. Conclusion

In this paper the stability behavior of turning processes with SSV was studied. The behavior of the properties of the time-varying delay dependent on a modulation of the spindle speed was investigated. A theory for the stability analysis of DDE, as a model for turning processes, in the limit of a slowly time-varying delay was presented. The present results of the frozen time approach give a connection between the stability results of constant speed machining and
variable speed machining. They are a counterpart to the results of (Michiels et al., 2005), which hold in the limit of high modulation frequencies of the time-varying delay. Whereas the approximations for high modulation frequencies are not very relevant for machining, the stability lobes of the frozen time approach approximate partially the numerically exact results for turning with an applicable kind of SSV. Furthermore, especially the connection between the stability of constant and variable speed machining makes it possible to understand the stabilizing mechanism, to estimate the stabilization capability and to develop strategies for an optimal parameter selection of the SSV.

In contrast to the results for milling processes (Otto et al., 2011), an implementation of a SSV in turning processes enables higher stable chip widths in comparison to constant speed machining. Furthermore, it was shown, that a large stabilization of the process is possible via the implementation of a suitable SSV. Due to the large stabilization capability of SSV in turning, there are no problems with temporarily large increased vibration amplitudes and the maximum acceleration of the spindle can be reduced to a minimum.

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