Stability Analysis of Machining with Spindle Speed Variation

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Abstract
In this paper the chatter stability of turning and full-immersion milling operations with spindle speed variation is studied. We present a method to calculate the stability lobes in the limit of very low and very high frequencies of the delay modulation. These approximations help to classify the results of numerically exact methods, as for example semi-discretization or multi-frequency approaches. For slowly time-varying delay, the position of the stability lobes is understandable from a simple connection between the lobes for constant and time-varying delay. Furthermore, this method can be used to estimate the efficiency of an application of spindle speed variation and helps to find optimal parameters for it.

1. INTRODUCTION
Since the pioneering works of Tlustý et al. [1] and Tobias [2] it is known, that quality, productivity and efficiency of metal cutting are often limited by unstable self-excited vibrations at the tool, the workpiece, the machine tool or parts of it, called regenerative chatter. A wavy outer surface of the chip causes dynamical variations of the cutting force, which again generates new waves on the inner surface of the chip. If this closed loop remains in an unstable area, the amplitude of the relative vibrations between tool tip and workpiece grows exponentially.

Stöferle and Grab [3] were able to show that spindle speed variation (SSV) during the cutting process can suppress unfavorable phase lags between inner and outer modulation of the chip and reduce chatter. Further studies showed experimentally that SSV mostly results in a stabilization of the machining operation. For turning we refer the reader for example to Sexton and Stone [4] or Al-Regib et al. [5]. Lin et al. [6] and Radulescu et al. [7] did extensive investigations on the effects of SSV in milling processes.

Therefore, SSV can be used to reach higher material removal rates just by a software modification of the CNC unit of the machine tool, which is an advantage in comparison to hardware modifications, such as modifications of the structure or the use of special cutting tools. On the other side, practical restrictions due to the limited spindle drive power and theoretical difficulties constrict an industrial application of SSV. Indeed, powerful methods to determine the stability of machining with SSV for a special configuration are available, but there are no reliable ways to estimate the maximum possible stability increase or to find optimal parameters for it.

An exact analytical way for calculating the stability for sinusoidal time-varying spindle speeds in the frequency domain was presented by Jayaram et al. for turning [8] and by Sastry et al. for milling [9]. Later, this approach was generalized by Zatarain et al. [10]. However, for practical implementation it is necessary to analyze a large number of frequencies. As a result, the so-called multifrequency approach is just another numerical method to calculate the stability of non-autonomous delay differential equation (DDE), such as for example semidiscretization [11].

The energy based analysis for finding optimal SSV parameters, which was presented in [5], considers only a single chatter frequency. It can be interpreted as a zeroth-order approximation and holds only for high frequencies of the SSV. For high modulation frequencies Michiels et al. proved in [12] that the stability of a DDE with rapidly time-varying delay is equal to the stability of the time-invariant distributed delay comparison system.

In this paper a simplified stability analysis of DDE with slowly time-varying delay, which was developed in [13], is used to investigate and
compare systematically the stability behavior of turning and milling with SSV. Furthermore the results are compared with the stability results of the distributed delay comparison system for high modulation frequencies.

The remainder of the paper is organized as follows. In section 2 the combined model for turning and full-immersion milling is defined, followed by Section 3, where different methods for the stability analysis are explained. In Section 4 the resulting stability lobes of the different methods are presented and interpreted and in Section 5 the theoretical results are verified by an experimental milling process with SSV.

2. MODELS

The model presented here is related to the milling model of Altintas [14]. It consists of an oscillator model of the structure, a linear cutting force model and the directional factor, which describes the interaction between cutting process and machine tool. By changing the directional factor it is possible to switch between the turning and the milling process.

2.1. Structural behavior

In [14] the dynamical behavior of the machine tool at the tool tip is modeled by a frequency response function (FRF). A system of uncoupled harmonic oscillators, whose parameters are determined via an evolutionary algorithm [15] can be fitted to the measured FRF. Since the stability of turning or milling is often limited only by a single mode of vibration in a predefined range of spindle speeds, the response of the structure can, in the simplest case, be described by a single harmonic oscillator

\[
\begin{align*}
\ddot{x} + 2\zeta\omega_n x + \omega_n^2 x &= F_x m^{-1}_x \\
\ddot{y} + 2\zeta\omega_n y + \omega_n^2 y &= F_y m^{-1}_y
\end{align*}
\]  

(1)

The \(x\)- and \(y\)-directions are defined in a fixed structural coordinate system, where \(x\) is in the direction of the feed motion (see Figure 1). The relative damping coefficients \(\zeta_{xy}\), the eigenfrequencies \(\omega_{xy}\) and the masses \(m_{xy}\) must be chosen such that the theoretical FRF of the harmonic oscillator in \(x\)- and \(y\)-direction coincides each with the highest peak of the measured FRF. The forces \(F_x\) and \(F_y\), acting on the structure, are a composition of the tangential and normal cutting force components \(F_t\) and \(F_n\) at the tool tip.

2.2. Cutting force

The nonlinear cutting force is linearized around the quasi-static chip thickness induced by the feed motion. Thus, the cutting force depends linearly on the dynamic chip thickness \(h(t)\), which is defined by dynamical displacements of the structure at the present and the previous cut

\[F_t = K_t h(t), F_n = K_n h(t) = \gamma F_t.\]  

(2)

The normal force \(F_n\) is in the direction of the uncut chip thickness and the tangential component \(F_t\) is in the direction of the cutting velocity (cf. Figure 1). The normal and tangential cutting force coefficients \(K_n\) and \(K_t\) must be identified experimentally for a special feed rate, fixed cutter as well as workpiece material and for high enough spindle speeds, where process damping can be disregarded. The ratio \(\gamma\) of normal versus tangential force component specifies the angle of the resulting cutting force. Portions of the cutting force in direction of the chip width \(b\), perpendicular to \(F_t\) and \(F_n\), were ignored.

2.3. Directional factor

The dynamical interaction between the structure and the cutting force is done by the directional factor \(D\)

\[D = \begin{pmatrix} d_{xx} & d_{xy} \\ d_{yx} & d_{yy} \end{pmatrix}.\]  

(3)

On the one hand \(D\) transforms the structural displacements at the present cut \(x(t), y(t)\) and at the previous cut \(x(t-\tau(t)), y(t-\tau(t))\) into displacements of the dynamic chip thickness \(h(t)\). On the other hand the directional factor implies also the transformation of the resulting dynamical cutting forces \(F_x\) and \(F_y\) back into forces \(F_t\) and \(F_n\) in the structural coordinate system. At first, we capture the dynamical displacements of the structure in the vector \(A\)

\[A = \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} x(t) - x(t-\tau(t)) \\ y(t) - y(t-\tau(t)) \end{pmatrix}.\]  

(4)
The time delay \( \tau(t) \) is equal to the time between two subsequent cuts and is constant for constant speed machining and time-varying for machining with SSV. With equations (2), (3) and (4) the cutting force vector \( \mathbf{F} \) in the structural coordinate system can be written in matrix form

\[
(F_x \quad F_y)^T = \mathbf{F} = K_t b \mathbf{D} \Delta.
\] (5)

The elements of the directional factor \( \mathbf{D} \) are defined by the process geometry. They contain the information of the cutting force directions for all \( z \) teeth of the cutter and depend on the angular positions \( \alpha_j \) of each tooth relative to the structural coordinate system (cf. [14]).

\[
\begin{align*}
\mathbf{d}_{xx} &= \sum_{j=0}^{z-1} -\frac{g(\alpha_j)}{2} [\gamma (1 - \cos 2\alpha_j) + \sin 2\alpha_j] \\
\mathbf{d}_{xy} &= \sum_{j=0}^{z-1} -\frac{g(\alpha_j)}{2} [(1 + \cos 2\alpha_j) + \gamma \sin 2\alpha_j] \\
\mathbf{d}_{yx} &= \sum_{j=0}^{z-1} -\frac{g(\alpha_j)}{2} [(\cos 2\alpha_j - 1) + \gamma \sin 2\alpha_j] \\
\mathbf{d}_{yy} &= \sum_{j=0}^{z-1} -\frac{g(\alpha_j)}{2} [\gamma (\cos 2\alpha_j + 1) - \sin 2\alpha_j]
\end{align*}
\] (6)

The function \( g(\alpha_j) \) specifies if tooth \( j \) is in the cut or not, with \( \alpha_{st} \) the start and \( \alpha_{ex} \) the exit angular positions to and from the cut.

\[
g(\alpha_j) = \begin{cases} 
1, & \alpha_{st} < \text{mod}(\alpha_j, 2\pi) \leq \alpha_{ex} \\
0, & \text{else}
\end{cases}
\] (7)

2.4. Dynamics of turning and milling

For a typical orthogonal turning process the number of teeth is one \( (z=1) \). The angular position of the tool is constant at \( \alpha_{t} = \pi/2 \) (cf. Figure 1) and it is permanently in the cut. Putting this in equation (6), we get the directional factor for turning

\[
\mathbf{D}_{\text{turn}} = \begin{pmatrix} -\gamma & 0 \\ 1 & 0 \end{pmatrix}.
\] (8)

As a result, vibrations in \( y \)-direction do not modify the cutting force, since the effect of these vibrations on the time delay [16] was neglected in our model. The \( y \)-motion of the tool is just a forced vibration, which cannot become unstable and the remaining dynamics in \( x \)-direction is equal to the traditional one dimensional turning model, studied for example in [4], [5], [8], [11] and [12].

In milling, there are typically more than one tooth and the angular positions \( \alpha_j \) of the teeth alternate due to the rotation of the tool. In general this results in a time-varying directional factor and cutting force. To compare the dynamics of milling with those of turning we use in this paper only the average component (zeroth-order approximation) of the directional factor.

\[
\mathbf{D}_{\text{mill}} = \begin{pmatrix} -\gamma & 1 \\ 1 & -\gamma \end{pmatrix}
\] (9)

This holds exactly for full-immersion cutting with \( z=4, 6, 8, \text{ etc.} \) teeth and is a good approximation, if there is permanently one or more than one tooth in the cut (cf. [14]).

Combining equation (5) with (8) or (9) and putting \( F_x \) and \( F_y \) into equation (1), we get a closed-loop model for regenerative chatter. By introducing the dimensionless time \( t' = \omega_{xy} t(2\pi) \) (cf. [11]), dropping the prime immediately and setting \( m_x = m_y \), the resulting system becomes independent of the absolute value of the eigenfrequencies \( \omega_{xy} \) and the masses of the structure as well as independent of the cutting force coefficient \( K_t \) and the chip width \( b \).

\[
\begin{pmatrix} \ddot{x} + 4\pi^2 \omega_x^2 x + 4 \pi^2 \varepsilon \dot{x} \\ \ddot{y} + 4 \pi^2 \omega_y \dot{y} + 4 \pi^2 \varepsilon^2 y \end{pmatrix} = w \mathbf{D} \Delta \mathbf{e} = \frac{\omega_y}{\omega_x} \frac{\mathbf{e}}{\varepsilon}
\] (10)

The normalized chip width \( w \) keeps the limiting depths of cut to avoid chatter as general as possible and is dimensionless.

\[
w = \frac{4 \pi^2 K_t b}{m_x \omega_x^2}
\] (11)

Mathematically, equation (10) is a system of DDEs with time-varying delay and physically it is a system of two coupled, self-excited harmonic oscillators.

3. STABILITY ANALYSIS

For the stability analysis of the DDE with a certain time-varying delay \( \tau(t) \) we use the semidiscretization method [11]. Furthermore, we analyze the limiting cases of a very slow and very fast delay modulation with the same amplitude. This will be helpful for classifying the degree of stabilization for a chosen amplitude and frequency of the SSV and helps to optimize the parameters with respect to a large stabilization due to the SSV.

3.1. Semidiscretization

Semidiscretization approximates the delay term of a DDE at discrete times \( t = i dt \), whereas the ODE part is left in its original form. For this pur-
pose we divide our machining model into an ODE and a DDE part.

\[ \dot{u}(t) = Au(t) + Bu(t - \tau(t)) \]
\[ u(t) = (x(t) \quad \dot{y}(t) \quad x(t) \quad y(t))^T \]  

(12)

Following [11] the discretization of the DDE part reduces the infinite dimensions of the continuous system to a finite number and the state space form of the DDE can be written in matrix-vector notation

\[ v_i = C_i v_{i-1}, \quad i = 1, 2, \ldots \]
\[ v_i = (u(t_i) \quad u(t_{i-1}) \quad \ldots \quad u(t_{i-M}))^T \]  

(13)

Even though, the directional factor \( D \), the cutting force \( F \), and consequently the matrices \( A \) and \( B \) are constant, the matrix \( C \) is time-variant because of the time-varying delay. The step size \( \Delta t \) must be chosen in such a way, that the number of time steps \( P \) for one period of the delay modulation \( T \) is an integer value.

\[ T = P \Delta t \]  

(14)

Then, the scalar value \( M \) is consistent with the number of steps for the maximum time delay rounded up to the next integer value. Thus, the size of the state space is \( 4^*(M+1) \) and \( v_{i-1} \) implies all past values of \( u \), which are able to influence the state vector \( v \) at the next time step. Floquet theory states that the transition matrix \( \Psi \) over the principle period \( T \), or \( P \) time steps, is a constant matrix. It serves as a finite dimension approximation of the monodromy operator

\[ v_P = \Psi v_0, \quad \Psi = CP_{P-1} \cdots C_1. \]  

(15)

The eigenvalues of \( \Psi \) are the characteristic multipliers. If the spectral radius \( \rho(\Psi) \) of the monodromy matrix \( \Psi \), which is equal to the modulus of the largest eigenvalue of \( \Psi \), is less than one, then system (12) is stable.

### 3.2. Low modulation frequencies

The chatter frequency \( \omega_c \), which defines the intrinsic time scale of the turning or milling process, is in the range of the eigenfrequencies \( \omega_x \) or \( \omega_y \) of the structure. In application the frequency of the delay modulation \( \omega = 2\pi/T \) is typically very low compared to \( \omega_c \). A separation of the intrinsic time scale \( \omega_c \) and the extrinsic time scale \( \omega \) is possible. Instead of analyzing the largest eigenvalue of the product \( \Psi \) of the matrices \( C \) over \( P \) time steps it is sufficient to multiply the largest eigenvalues of the matrices \( C \) over the principal period.

\[ \rho(\Psi) = \prod_{i=1}^{P} \rho(C_i), \quad \text{for } \omega << \omega_c \]  

(16)

This is called frozen time approach (see [13] for details) since the extrinsic time scale of the delay variation is frozen. The stability of the system with time-varying delay can approximately be inferred from the stability behavior of the time-invariant system at frozen delays \( \tau(t) \).

Equation (16) holds for \( \omega << \omega_c \), because in this case the system matrix \( C \) can be considered as time-invariant over one state interval \([t; t+M] \).

As a result, the dominant intrinsic frequency \( \omega_c \) of the solution \( v \) of equation (12) does not change significantly with time, the transient behavior of the solution is negligible and their exponential behavior can be specified by the evolution of the modulus of the largest eigenvalue of the system matrices \( \rho(C) \).

### 3.3. High modulation frequencies

For the unrealistic case of a very high frequency \( \omega \) of the delay modulation relative to the chatter frequency \( \omega_c \) the time delay adopts all values of the delay variation in one period of the intrinsic time scale. In [12] Michiels et al. relate the stability analysis of equation (12) with a fast time-varying point-wise delay to the analysis of the comparable time-invariant DDE with distributed delay. The stability of the distributed delay comparison system is determined by the roots of the characteristic equation.

\[ \det(sI - A - Be^{-s\tau_0} k(s\delta)) = 0 \]  

(17)

The term \( k(s\delta) \) is a correction term due to the distributed delay and \( \delta \) is the amplitude of the delay modulation around the nominal delay \( \tau_0 \).

\[ \tau(t) = \tau_0 + \delta f(\omega t) \]  

(18)

Dependent on the function \( f(\omega t) \) of the delay modulation \( k(s\delta) \) has different forms (see [12] for details). If there is no delay variation, the correction term disappears to \( k(0)=1 \) and equation (17) is simply the characteristic equation of the system with constant delay \( \tau_0 \).

With the Laplace transformation of the cutting force of equation (5) and the structural model of equation (1) we get the characteristic equation (17) for our special model.

\[ \det(I - K_j b(1 - k(s\delta)e^{-s\tau_0}) \Phi(s)) = 0 \]  

(19)

The structural behavior in the frequency domain is summarized in a matrix \( \Phi(s) \) containing the transfer functions \( \Phi_{xy}(s) \) of the structure in \( x \) - and \( y \)-direction.
By setting $s = \tilde{k}_0\epsilon$, the stability lobes can be calculated similar to [14] with an additional complexity due to the correction term $k(s\tilde{\epsilon})$, which will be not presented in this paper.

### 4. THEORETICAL RESULTS

In this section the stability results are illustrated at first for the turning model and later for the milling model for different parameters of the structure and the delay modulation. The lobes for constant delay and for distributed delay were calculated analytically by solving equation (19). For the low frequency approximation semidiscretization was used to calculate the stability behavior $\rho(C)$ for the different frozen delays. For delay modulations with a certain frequency $\nu = \omega/(2\pi)$ the stability lobes were computed via time domain simulations by the DDE solver RADAR5 (see [17]) and verified randomly by semidiscretization.

#### 4.1. Turning

Putting equation (8) into equation (10), we get the traditional turning model for regenerative chatter.

$$\ddot{x} + 4\pi^2 \gamma x + 4\pi^2 x = w_\nu(x(t-\tau(t)) - x(t))$$

(21)

The ratio $\gamma$ is set to be one, since it does not change the stability results qualitatively. Hence, the normalized chip width $w$ depends only on the relative damping coefficient $\gamma$, and on the characteristics of the delay modulation. In Figure 2 the limiting chip width $w(\tau_0)$, where stability is lost, is plotted over the nominal delay $\tau_0$ for $\gamma = 0.03$. The shape of the delay variation $\nu/(2\pi\nu)$ is a sawtooth function, defined as in [12]. Stability lobes are shown for the amplitude $\delta = 0.5$ and different frequencies $\nu$ of the delay modulation. They are compared with the limiting cases of a very high ($\nu \rightarrow \infty$) and a very low frequency ($\nu \rightarrow 0$) of the delay modulation and with the lobes for constant delay ($\delta = 0$).

For $w = 0$ the turning process is stable. After crossing a stability lobe by increasing $w$, the process becomes unstable for the corresponding cutting conditions. For larger values of $\nu$ an unstable process can be also re-stabilized by further increase of $w$ (see the curve for $\nu = 0.3$). An increase of the stability boundary by a factor of ten or more compared to the process with constant delay is possible due to a fast delay variation.

Theoretically an arbitrarily small frequency $\nu$ is sufficient to increase the minimum limiting depth of cut and reach the stability border for $\nu \rightarrow 0$. However, the process over the principle period $T$ can be globally stable, but locally unstable, if $T$ is much larger than the machining time (cf. [18]). Therefore, from a practical point of view $\nu$ must be high enough, to pass a local unstable cutting condition fast enough (see [13] for details).

In turning with SSV a destabilizing effect of the delay modulation can occur only for very small frequencies of the delay modulation. In Figure 2 basically only the lobe for $\nu = 0.002$ intersects with the peaks of the lobe for constant delay at small values of the nominal time delay $\tau_0$. For small $\tau_0$ and very low $\nu$ the transient behavior of the vibrations due to a slight change of the time delay is negligible. The resulting stability of the process with time-varying delay is composed of the individual stability behavior of the process with constant delays.

#### 4.2. Milling

In milling also the cutting force ratio $\gamma$ as well as the structural behavior in $y$-direction, which is specified by the relative damping coefficient $\gamma$ and the ratio $\epsilon$ of the eigenfrequencies $\omega_y$ and $\omega_x$, influence qualitatively the stability of the process.

In Figure 3 the stability behavior is shown for a milling process with $\gamma = 0.03$, $\gamma = 0.02$, $\epsilon = 1.1$ and $\gamma = 0.4$ for constant time delay. Furthermore, stability lobes are presented for a sinusoidal time-varying delay with an amplitude $\delta = 0.4$ and
different frequencies \( \nu \) of the delay modulation. The lobes for a certain frequency \( \nu \) of the delay modulation are either more related to the lobes of the limiting cases of a very high (\( \nu \to \infty \)) or a very low frequency (\( \nu \to 0 \)) of the delay modulation. Hence, at first we want to discuss the resulting stability lobes for the distributed delay comparison system (\( \nu \to \infty \)), because for a milling process to our knowledge no comparable results were presented anywhere else. For turning Michiels et al. showed in [12] that for any \( \delta > 0 \) the distributed delay comparison system is always more stable than the system with constant delay (\( \delta = 0 \)). This is closely related to the fact that the imaginary part of the roots \( \omega_c \) on a stability lobe for constant delay always decrease for increasing nominal delay \( \tau_0 \). In milling the imaginary part of the root of equation (19) with zero real part \( \omega_c \) can also increase for increasing nominal delay \( \tau_0 \). As a result it is possible, that a milling process with fast time-varying delay is more unstable than the same process with constant delay (see e.g. Figure 3 for \( \tau_0 \sim 2 \)). As a second difference, for milling the minimal limiting depth of cut of the distributed delay comparison system (\( \nu \to \infty \)) is approximately only twice as high as the minimal limiting depth of cut for the system with constant delay (\( \delta = 0 \)) whereas for turning much larger stabilization gains are possible. Thirdly, in milling for \( \omega_c > \omega_y \) no re-stabilization or instability islands appear in contrast to turning. The stability lobes in Figure 3 for milling with sinusoidal time-varying delay for the frequencies \( \nu = 0.5 \), \( \nu = 0.02 \), \( \nu = 0.001 \) and \( \nu \to 0 \) are qualitatively not different from the results of turning (Figure 2). The stability borders for higher modulation frequencies behave similar to the associated limiting case of a distributed delay and for low frequencies similar to the resulting stability lobes of the frozen time approach, which is just the average stability behavior of the subsequent time-invariant cutting conditions. Note also that the stability lobes of a very high (\( \nu \to \infty \)) and a very low (\( \nu \to 0 \)) modulation frequency are not the upper and the lower bound on the stability lobes for any frequency \( \nu \) of the delay modulation as it is illustrated in Figure 4 with the parameters \( \zeta_x = 0.03 \), \( \zeta_y = 0.02 \), \( \epsilon = 1.2 \), \( \gamma = 0.2 \) and \( \delta = 0.4 \). For \( \nu = 1 \) the milling process can be stable, while the system for \( \nu \to \infty \) is unstable and for \( \nu = 0.1 \) the milling process can be unstable, while the system for \( \nu \to 0 \) is stable. This is due to an extremely favorable or possibly unfavorable relationship of \( \nu \) and \( \tau_0 \). Nevertheless, the stability lobes of the limiting cases determine the range of the stability lobes for any modulation frequency and are reliable for estimating the minimal and maximal possible stabilization due to a variation of the time delay.

5. EXPERIMENTAL RESULTS

The theoretical results for the stability of machining with spindle speed variation are verified by an experimental determination of the stability lobes. For this purpose, cutting tests were done on a Heller machining center H2000.

5.1. Parameter adjustment

The cutting configuration was full-immersion end milling with a feed rate of 0.15mm feed per tooth. The workpiece was AlMg4.5Mn0.7 and the tool a four-fluted (z=4) Gühring PKD milling cutter with a diameter of 24mm.
A FRF of the structure was measured at the tool tip by an impact hammer test and mapped by the method of [15] to a system of harmonic oscillators in the time domain. The dominant eigenfrequencies of the structure in x- and y-direction were close to each other at \( \omega_x=7634.07 \text{s}^{-1} \) and \( \omega_y=7728.32 \text{s}^{-1} \), respectively. The whole structural model includes eleven oscillators in x- and ten oscillators in y-direction.

With the structural model the stability lobes of the selected milling process for constant delay can be calculated for different cutting force coefficients. The cutting force coefficients \( K_t=350 \text{N/mm}^2 \) and \( \gamma=0.2 \) for the experimental set-up were determined by a fit of the numerically computed stability lobes to experimental data for the stability of the milling process with constant spindle speed. In the experiments the limiting depth of cut, which separates chatter vibrations from chatter free cutting, was identified acoustically by a microphone.

5.2. Spindle speed variation

In practical milling processes a variation of the spindle speed results in a time-varying delay between two subsequent cuts.

\[
\Omega(t) = \Omega_0 + \delta_\Omega f(\omega t) \tag{22}
\]

The exact relation between spindle speed \( \Omega(t) \) (in RPM) and time-delay \( \tau(t) \) was given for example in [11]

\[
\int_{\tau(t)}^{\tau(t)+60} \Omega(t') \, dt' = \frac{1}{z}. \tag{23}
\]

Approximations of the shape of the time-varying delay from a given SSV must be done carefully within a specific parameter range. It is not possible to reach any kind of delay modulation by an arbitrary SSV. A more detailed study of the relation between spindle speed and delay was done in [13].

Another restriction on the possible delay modulations is the limited spindle drive power. Since the maximum reachable frequency of the SSV is still small against the eigenfrequencies of the structure and the process with this maximum modulation frequency is not significantly more stable than the process with a lower modulation frequency (cf. the results of section 4.2) we choose a frequency \( \nu=2 \text{Hz} \) for our experiment, which is just high enough to prevent large temporal chatter vibrations (cf. section 4.1). Besides, \( \nu=2 \text{Hz} \) is small against the range of spindle speeds and the time delay can be approximated by

\[
\tau(t) \approx \frac{60}{\Omega_0 + \delta_\Omega f(\omega t)}. \tag{24}
\]

In the experiment the amplitude of the SSV is chosen to be proportional to the nominal value of the spindle speed \( \delta_\Omega=0.15 \Omega_0 \).

5.3. Stability results

The stability results of the milling experiments are presented in Figure 5. At first the black curve with square symbols (\( \delta_\Omega=0 \), exp.) was measured by the acoustic test. The green curve with point symbols (\( \delta_\Omega=0 \), sim.) was calculated with the measured structural data and the fitted cutting force coefficients. After that, the red curve with plus symbols (\( \delta_\Omega=0.15 \Omega_0 \), \( \nu=2 \text{Hz} \), sim.) was calculated, which predicts the stability lobes of the process with SSV. The blue curve with circle symbols (\( \delta_\Omega=0.15 \Omega_0 \), \( \nu=2 \text{Hz} \), exp.) is the experimentally determined stability border.

The stabilization gain of the minimal limiting depth of cut with SSV is up to 50\% compared to the minimal limiting depth of cut with constant spindle speed. The experimental results for the stability border are consistent with the theoretical predicted results of the simulation, even though we use only a simple linear model. The theoretical stability lobes again agree very well with the stability behavior of the limiting case of a very low modulation frequency (\( \delta_\Omega=0.15 \Omega_0 \), \( \nu=0 \text{Hz} \), approx.). This is due to the fact that the frequencies in both structural directions are very close to each other, which reduces transient behavior. Note that the frozen time approximation for very low modulation frequencies was done only with a reduced structural model with two modes in x- and three modes in y-direction.

![Figure 5: Stability lobes for the experimental milling process with sinusoidal time-varying spindle speed.](image_url)
6. CONCLUSION

Chatter stability was determined for turning as well as for milling processes with a time-invariant directional factor and time-varying delay. The presented results for the limiting cases of very low and very high frequencies of the delay modulation are suited to classify the previously often unexplained results for different parameters of the delay modulation. Especially the frozen time approach for slowly time-varying delay is a suitable tool for preparing an application of variable speed machining and helps to estimate the practically reachable stabilization gain with respect to constant speed machining. Furthermore, the frozen time approach can be used to optimize the parameters of the SSV with respect to a maximal stabilization if the dominant eigenfrequencies of the structure in x- and y-direction are close to each other.

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