Universität Kaiserslautern
Prof. Dr. C. Helmberg, B. Guta

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Nonlinear Programming
Exercises 7

1. In a circuit with impressed voltage $\varepsilon(t)$ and inductance L, Kirchhoffs's first law gives the relationship

$$\varepsilon(t) = L\frac{di}{dt} + Ri(t)$$

where R is the resistance in the circuit and i the current. Suppose we measured the current $i(t)$ for several values t and obtain:

t	1.00	1.01	1.02	1.03	1.04
i	3.10	3.12	3.14	3.18	3.24

where t is measured in seconds, i in ampers, the inductance L is a constant 0.98 henries, and the resistance is 0.142 ohms.

   a) Determine the voltage $\varepsilon(t)$ at each of the given time t using the central-difference formula and an appropriate one-sided difference formula at the left and right end points.

   b) Determine $\frac{d\varepsilon}{dt}$ at the values t=1.005, 1.015, 1.025, and 1.035 using the central difference formula.

2. Suppose the values of a function f contain roundoff errors u. Show that a suitable value for the perturbation ϵ in the centeral-differnce formula is approximately $\epsilon = u^{1/3}$, and that the accuracy achievable by this formula is approximately $u^{2/3}$. [Use similar assumptions to the ones used in the lecture to derive the estimates of ϵ for the forward-difference formula.]

3. Let $f(x,y) = \arctan(\frac{y}{xe^x})$. Using the forward and central-difference fromulas approximate the value of the gradient and hessian matrix of f at $(x,y) = (3,4)$ with $\epsilon = 10^{-5}$. Compare the results with the corresponding values that can be obtained analytically.

4. Suppose the Jacobian of a function $f : \mathbb{R}^8 \longrightarrow \mathbb{R}^6$ has the following structure:

x	x	x					
x		x	x				
	x	x				x	
		x	x	x			
					x	x	x
				x	x		x

where each row corresponds to the gradient of one coordinate function of f, each cross represents a nonzero element, and zeros are represented by blank space. Construct the graph structure that can be used to determine the full Jacobian, and find a valid coloring scheme with three colors. Are the three colors the minimum? Why?