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Nonlinear Programming
Exercises 5

1. Consider the one dimensinal function $\phi(\alpha) = f(x_k + \alpha p_k)$ where $\alpha > 0$ and p_k is a descent direction of a function $f \in C^1(\mathbb{R}^n)$ at x_k.

   a) Show that the quadratic function that interpolates $\phi(0), \phi'(0)$ and $\phi(\alpha_0)$, for some $\alpha_0 > 0$, is given by

   $$\phi_q(\alpha) = \Big(\frac{\phi(\alpha_0) - \phi(0) - \alpha_0\phi'(0)}{\alpha_0^2}\Big)\alpha^2 + \phi'(0)\alpha + \phi(0).$$

   b) Show that if the Armijo condition is not satisfied at α_0, then this quadratic function, ϕ_q , has positive curvature and that its minimizer, α_1, satisfies,

   $$\alpha_1 < \frac{1}{2(1-c_1)}.$$

   where c_1 is the constant in the Armijo condition.
   (Since c_1 is chosen to be quite small in practice, this indicates that α_1 cannot be much greater than $\frac{1}{2}$ (and may be smaller), which gives us an idea of the new step length.)

2. Suppose that the Armijo condition is not satisfied at the step lenghts α_0 and α_1, and consider the cubic interpolating $\phi(0), \phi'(0), \phi(\alpha_0)$ and $\phi(\alpha_1)$, where ϕ is given as in the problem 1.

   a) By drawing graphs illustrating the two situations that arise, show that the minimizer of the cubic lies in $[0, \alpha_1]$.

   b) Show that if $\phi(0) < \phi(\alpha_1)$, the minimizer is less than $\frac{2}{3}\alpha_1$.

3. Let $f(x) = 10(x_2 - x_1^2)^2 + (1 - x_1)^2$. At $x = (0, -1)$ draw the contour lines of the quadratic model

   $$m(p) = f(x) + \nabla f^T(x)p + \frac{1}{2}p^T Bp,$$

   assuming that B is the Hessian of f. Draw the family of solutions of the subproblem

   $$\min_{p\in\mathbb{R}^n} m(p) = f(x) + \nabla f(x)p + \frac{1}{2}p^T Bp, \qquad s.t.\ \ \|p\| \le \Delta,$$

   as the trust region radius varies from $\Delta = 0$ to $\Delta = 2$. Repeat this at $x = (0, 0.5)$.

4. Theorem III.2, in the Lecture, shows that the sequence $\{\nabla f_k\}$ has an accumulation point at zero. Show that if the iterates x stay in a bound set $\mathcal{B}$, then there is a limit point x_∞ of the sequence $\{x_k\}$ such that $\nabla f(x_\infty) = 0$.