Universität Kaiserslautern
Prof. Dr. C. Helmberg, B. Guta

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Nonlinear Programming
Exercises 2

1. Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ and $f \in C^1$. A vector $d \in \mathbb{R}^n$ is called a *direction of descent* of f at $\bar{x}$ if there exists a $T > 0$ such that
$$f(\bar{x} + td) < f(\bar{x}) \quad \text{for each} \quad t \in (0, T).$$
   a) Show that if $\nabla f^T(\bar{x})d < 0$, then d is a descent direction.
   b) Let $f(x) = x_1^2 \cos(x_1 - 1) + x_2^2$. Show that $d = (0, -1)^T$ is a descent direction of f at $\bar{x} = (1, 1)$ and determine the corresponding T.

2. Determine and classify the stationary points of the following functions with respect to necessary and sufficient conditions.
   a) $f_1(x) = x_1^3 - 2x_1 - x_1^2 + 6x_2 + x_2^2$
   b) $f_2(x) = x_2^2 e^{x_1} + e^{(x_2 - x_1)} + x_1 x_2 + x_3$
   c) $f_3(x) = 3x_2 - 2x_1^2 - 6x_2^2 - 4x_1$

3. Consider the unconstrained quadratic problem
$$\begin{array}{l} \min \;\; x^T Q x + b^T x \\ x \in \mathbb{R}^n \end{array}$$
where $Q \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$. Derive the first and second order necessary conditions as well as the sufficient conditions for $x^* \in \mathbb{R}^n$ to be a solution of the problem.

4. Apply Newton's method to the following functions, starting from the given initial point x_0 and display the iterates in a contour plot (see the example plot `www.mathematik.uni-kl.de/~helmberg/nlp_s02/exercise2_4.m`).
   a) $f_1(x) = \sqrt{x^2 + y^2 + 1} \; + \; y \sin(y)$,
   (i) $x_0 = (-1, -6)$, (ii) $x_0 = (1, 6)$.
   b) $f_2(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$,
   (i) $x_0 = (1.2, 1.2)$, (ii) $x_0 = (-1.2, 1)$.

5. Suppose Newton's method is applied to $f(x) = \cos(x)$.
   a) Find a starting point so that (in theory) the sequence of iterates diverges.
   b) Find a starting point so that the iterates alternate between two non optimal points.
   c) If the procedure starts from a point near 0, then to what point does the method converge? Why?
   d) Determine the maximal interval containing π so that for all starting points in this interval the iterates converge to π.