Universität Kaiserslautern
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Nonlinear Programming
Exercises 13

1. Consider the QP

$$\begin{array}{ll} \min & 2x_1^2 + x_2^2 + x_3^2 - 2x_1x_3 \\ s.t. & -2x_1 + x_2 \geq 2 \\ & -x_2 - x_3 \geq -2. \end{array}$$

   a) Show that the objective function is strictly convex.
   b) Formulate the dual of the problem in terms of only the dual variables, λ.
   c) Solve the dual problem applying the gradient projection method and compute the optimal solution of the primal problem x^* from the optimal solution of the dual problem λ^*.

2. The problem of finding the shortest distance from a point x_0 to the hyperplane $\{x : Ax = b\}$ where A has full row rank can be formulated as the quadratic program

$$\begin{array}{ll} \min & \frac{1}{2}(x - x_0)^T(x - x_0)^T \\ s.t. & Ax = b. \end{array}$$

   Show that the optimal Lagrange multiplier is

$$\lambda^* = (AA^T)^{-1}(b - Ax_0),$$

   and that the solution is

$$x^* = x_0 - A^T(AA^T)^{-1}(b - Ax_0).$$

   Show that in the special case where A is a row vector, the shortest distance from x_0 to the solution set of $Ax = b$ is

$$\frac{|b - Ax_0|}{||A||}.$$

3. Program Algorithm XI.3 (Active-set Method) and use it to solve

$$\begin{array}{ll} \min & x_1^2 + 2x_2^2 - 2x_1 - 6x_2 - 2x_1x_2 \\ s.t. & \frac{1}{2}x_1 + \frac{1}{2}x_2 \leq 1, \\ & -x_1 + 2x_2 \leq 2, \\ & x_1, x_2 \geq 0. \end{array}$$

   Choose three initial starting points: one in the interior of the feasible region, one at a vertex, and one on the boundary of the feasible region(not a vertex).

4. Write a program that implements Algorithm XII.2 (Local SQP Algorithm). Use it to solve the problem

$$\begin{array}{ll} \min & e^{x_1x_2x_3x_4x_5} - \frac{1}{2}(x_1^3 + x_2^3 + 1)^2 \\ s.t. & x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0 \\ & x_2x_3 - 5x_4x_5 = 0, \\ & x_1^3 + x_2^3 + 1 = 0. \end{array}$$

   Use the starting point $x_0 = (-1.71, 1.59, 1.82, -0.763, -0.763)$.
   (The solution is $x^* = (-1.8, 1.7, 1.9, -0.8, -0.8)$).