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Nonlinear Programming
Exercises 12

1. Consider the minimization problem:

$$\min \tfrac{1}{1+x^2}, \qquad \text{subject to} \quad x \geq 1.$$

Write down the logarithmic barrier function $P(x;\mu)$ for this problem, and show that $P(x,\mu)$ is unbounded below for any positive value of μ.

2. Consider the scalar minimization problem:

$$\begin{array}{ll} \min & x \\ s.t. & x^2 \geq 0 \\ & x+1 \geq 0, \end{array}$$

for which the solution $x^* = -1$. Write down the logarithmic barrier function $P(x;\mu)$ for this problem and find its local minimizers. Show that for any sequence $\{\mu_k\}$ such that $\mu_k \downarrow 0$, there exists a corresponding sequence of local minimizers $x(\mu_k)$ that converges to 0.

3. Consider the following problem:

$$\begin{array}{ll} \min & (x_1-5)^2 + (x_2-3)^2 \\ s.t. & x_1 + x_2 \leq 3 \\ & -x_1 + 2x_2 \leq 4, \end{array}$$

Formulate a suitable barrier problem with the initial parameter $\mu = 1$. Use an unconstrained optimization technique starting with the point $x^0 = (0.0)^T$ to solve the barrier problem.

4. When a minimization problem formulation contains inequality constraints, the augumented Lagrangian method yields the follwing subproblem to be solved at each iteration k :

$$\begin{array}{ll} \min_{x,s} & f(x) - \sum_{i\in\mathcal{I}} \lambda_i^k (c_i(x) - s_i) + \frac{1}{2\mu_k} \sum_{i\in\mathcal{I}} (c_i(x) - s_i)^2 \\ s.t. & s_i \geq 0, \quad \forall i \in \mathcal{I}. \end{array}$$

   a) Show that the optimal values of the s_i are

$$s_i = \max\{c_i(x) - \mu\lambda_i^k, 0\}, \qquad \forall i \in \mathcal{I}.$$

   b) By defining the function

$$\psi(t,\sigma;\mu) = \begin{cases} -\sigma t + \frac{1}{2\mu} t^2 & \text{if } t - \mu\sigma \leq 0, \\ -\frac{\mu}{2}\sigma^2 & \text{otherwise,} \end{cases}$$

   show that we obtain the transformed subproblem

$$\min_x \mathcal{L}_A(x, \lambda^k; \mu_k) = f(x) + \sum_{i\in\mathcal{I}} \psi(c_i(x), \lambda_i^k; \mu_k),$$

   and argue that the Lagrange multipliers should be updated by

$$\lambda_i^{k+1} = \max\{\lambda_i^k - c_i(x_k)/\mu_k, 0\}, \quad \forall i \in \mathcal{I}.$$