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Nonlinear Programming
Exercises 11

1. Consider the matrix

$$\begin{bmatrix} \nabla^2 f(x^*) - \lambda^{*T}\nabla^2 c(x^*) & -\nabla c(x^*)^T \\ \nabla c(x^*) & 0 \end{bmatrix}$$

   constructed in the proof of Theorem IX.13, in the lecture. Show that under the assumptions of the theorem the matrix is nonsingular.

2. a) Consider the convex program

$$\begin{array}{ll} \min & f(x) \\ s.t & c_i(x) = 0, \quad i \in \mathcal{E} \\ & c_i(x) \geq 0, \quad i \in \mathcal{I} \end{array}$$

   where $f(x)$ is convex, $c_i(x)$ for $i \in \mathcal{E}$ are linear and $c_i(x)$ for $i \in \mathcal{I}$ are concave functions. Show that the feasible set of the convex program is convex.

   b) Show that the program

$$\begin{array}{ll} \min & x^2 - 6x + y^2 + 2y \\ s.t & x + 2y - 10 = 0, \\ & -x^2 - y^2 + 25 \geq 0 \end{array}$$

   is a convex program and solve the problem.

3. Prove Corollary X.3, in the lecture.

4. A new facility is to be placed such that the sum of its squared distance from four existing facilities is minimized. The four facilities are located at the points (1,2), (-2,4), (2,6), and (-6,-3). If the coordinates of the new facility are x_1 and x_2, suppose that x_1 and x_2 must satisfy the restrictions $x_1 + x_2 = 2, x_1 \geq 0$ and $x_2 \geq 0$.

   a) Formulate the problem.
   b) Show that the problem is a convex program.
   c) Find an optimal solution by making use of the KKT conditions.
   d) Solve the problem by a penalty function method using a suitable unconstrained optimization technique.

5. Consider the problem minimize x^3 subject to $x - 1 = 0$. Note that $x^* = 1$ is the optimal solution. Now consider the problem to minimize $x^3 + \mu(x-1)^2$.

   a) For $\mu = 1, 10, 100$, and 1000, plot $x^3 + \mu(x-1)^2$ as a function of x, and for each case, find the point where the derivative of the function vanishes. Also, verify that the optimal solution is unbounded.
   b) Show that the optimal solution to the penalty problem is unbounded for any given μ, so that the conclusion of Theorem X.2, in the lecture, does not hold. Discuss.
   c) For $\mu = 1, 10, 100$, and 1000, find the optimal solution to the penalty problem with the added constraint $-|x| + 2 \geq 0$.