Universität Kaiserslautern
Prof. Dr. C. Helmberg, B. Guta

Kaiserslautern, June 25, 2002
Date of Submission: Juli 2, 2002

Nonlinear Programming
Exercises 10

1. Let $\Omega \subseteq \mathbb{R}^n$ be a non-empty convex set and $f : \Omega \longrightarrow \mathbb{R}^n$ be a differentiable convex function. Consider
$$\min_{x\in\Omega} f(x).$$
Show that $\bar{x} \in \Omega$ is an optimal solution iff $\nabla f(\bar{x})^T(x-\bar{x}) \geq 0 \quad \forall x \in \Omega$.

2. Consider
$$\begin{array}{ll} \min & (x_1 - 3/2)^2 + (x_2 - 5)^2 \\ s.t. & -x_1 + x_2 \leq 2 \\ & 2x_1 + 3x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{array}$$
   a) Graph the feasible set .
   b) Solve the problem graphically and prove the optimality of the solution using Problem (1).
   c) Show that $\hat{x} = (0,0)^T$ is not optimal. Determine a feasible improvement direction and a better solution.

3. Consider the problem
$$\begin{array}{ll} \min & x_1x_2 + x_2x_3 + x_1x_3 \\ s.t. & x_1 + x_2 + x_3 = 3. \end{array}$$
   a) Write down the KKT-conditions for this problem and find all the solutions satisfying the condition.
   b) Write down the second-order sufficient conditions for this problem and determine whether a solution you found in (a) is optimal.

4. Prove that when the KKT conditions (Lemma IX.9) and the LICQ are satisfied at a feasible point x^*, the Lagrange multiplier vector λ^* of Lemma IX.9 is unique.