Exercise sheet 4 – released on 26.11.2019

Exercise 4.1 (voting Exercise)
Let $\mathbf{A}(\mu) \in \mathbb{R}^{H\times H}$ be the system matrix of a discrete detailed problem, $\mathbf{K} \in \mathbb{R}^{H\times H}$ the inner product matrix of X and $\mathbf{A}_s(\mu) := \frac{1}{2}(\mathbf{A}(\mu) + \mathbf{A}(\mu))^\mathsf{T}$ the *symmetric part* of $\mathbf{A}$. Show that the coercivity constant can be obtained as follows

$$\alpha(\mu) = \lambda_{\min}(\mathbf{K}^{-1}\mathbf{A}_s(\mu))$$

where $\lambda_{\min}$ denotes the smallest eigenvalue.

Exercise 4.2 (written Exercise)[4 Points]
Let $a(\cdot,\cdot;\mu)$ be parameter separable where the components are symmetric and satisfy $a_q(u,u) \geq 0$ and coefficient functions satisfy $0 < \theta_q^a(\mu) < \infty$, for $q = 1,\dots,Q_a$ and all $u \in X$, $\mu \in \mathcal{P}$. Furthermore, let $\bar{\mu} \in \mathcal{P}$ such that $\gamma(\bar{\mu})$ is available. Show that

$$\gamma(\mu) \leq \gamma_{UB}(\mu) < \infty, \quad \forall \mu \in \mathcal{P},$$

with the upper bound

$$\gamma_{UB}(\mu) := \gamma(\bar{\mu}) \cdot \max_{q=1,\dots,Q_a} \frac{\theta_q^a(\mu)}{\theta_q^a(\bar{\mu})}.$$

Exercise 4.3 (voting Exercise)
Let $\{u_i\}_{i=1}^n \subset X$. Show that the *Gramian matrix*

$$\mathbf{K} := \left(\langle u_i, u_j\rangle_X\right)_{i,j=1}^n \in \mathbb{R}^{n\times n}$$

has the following properties:

(a) $\mathbf{K}$ is symmetric and positive semidefinite.

(b) $\operatorname{rank}(\mathbf{K}) = \dim\left(\operatorname{span}(\{u_i\}_{i=1}^n)\right)$.

Exercise 4.4 (programming Exercise)[4 Points]
Find on the webpage the file `Sheet4_Data.mat`. It contains a `model`, `model_data`, `detailed_data` and `reduced_data` for the Thermal Block with 3×3 blocks using a Lagrange reduced basis for the parameter vectors $\mu(i)$, $i \in \{0.1, 0.5, 0.9, 1.3, 1.7\}$, where the function $\mu(\cdot)$ is defined as

$$\mu : \mathbb{R} \to \mathbb{R}^9 : i \mapsto (i,1,1,1,1,1,1,1,1)^\mathsf{T}.$$

Based on this function we define the functions

$$e(i) := \|u(\mu(i)) - u_N(\mu(i))\|_X, \quad \Delta(i) := \Delta_N(\mu(i)),$$
$$\eta(i) := \frac{\Delta(i)}{e(i)}, \quad b(i) := \frac{\gamma(\mu(i))}{\alpha(\mu(i))}.$$

Evaluate and visualize these functions for $i \in [0.01, 5]$ and discuss which theoretical results can be seen in these plots.
Tip: After `rb_sim_data = rb_simulation(model, reduced_data)` the error estimator can be accessed via `rb_sim_data.Delta`.

Explanation of terms

- For **written Exercises** hand in your elaboration/solution after one of the lectures or via Email to Dominik Garmatter until **09.12.2019**.
- For **programming Exercises** send your **commented** solution in `MATLAB`-Code via E-mail to Dominik Garmatter. Please, start the subject of your E-mail with **"MOR_201920_4:"**.
- For **voting Exercises** no written solution is requested. The solution for these exercises will be discussed/presented during the exercise course.
- All exercises of Exercise sheet 4 will be discussed during the exercise on 10.12.2019.