TECHNISCHE UNIVERSITÄT CHEMNITZ

Exercise sheet 3 – released on 12.11.2019

Exercise 3.1 (voting Exercise)

Prove Proposition 1.27: let the bilinear form a and the linear forms f, l fulfill the assumptions of the detailed problem and in addition let a, f, and l be Lipschitz-continuous w.r.t. μ with Lipschitz-constants L_a, L_f, and L_l. Then,

$$\|u(\mu_1) - u(\mu_2)\|_X \leq C_1 \|\mu_1 - \mu_2\|_2, \qquad \|u_N(\mu_1) - u_N(\mu_2)\|_X \leq C_1 \|\mu_1 - \mu_2\|_2,$$
$$|s(\mu_1) - s(\mu_2)| \leq C_2 \|\mu_1 - \mu_2\|_2, \qquad |s_N(\mu_1) - s_N(\mu_2)| \leq C_2 \|\mu_1 - \mu_2\|_2,$$

for all $\mu_1, \mu_2 \in \mathcal{P}$ and Lipschitz-constants $C_1 := \frac{L_f}{\bar{\alpha}} + \frac{L_a \bar{\gamma}_f}{\bar{\alpha}^2}$ as well as $C_2 := L_l \frac{\bar{\gamma}_f}{\bar{\alpha}} + \bar{\gamma}_l C_1$.

Exercise 3.2 (written Exercise)[4 Points]

Prove Proposition 1.45: for all $\mu \in \mathcal{P}$ we have the effective error estimator for the error in the energy norm via

$$\|u(\mu) - u_N(\mu)\|_\mu \leq \Delta_N^{en}(\mu) := \frac{\|v_r(\mu)\|_X}{\sqrt{\alpha_{LB}(\mu)}} \quad \text{and} \quad \eta_N^{en}(\mu) := \frac{\Delta_N^{en}(\mu)}{\|u(\mu) - u_N(\mu)\|_\mu} \leq \sqrt{\frac{\gamma_{UB}(\mu)}{\alpha_{LB}(\mu)}}.$$

Furthermore, if for all $\mu \in \mathcal{P}$

$$\Delta_N^{en,rel}(\mu) := \frac{2}{\|u_N(\mu)\|_\mu} \cdot \frac{\|v_r(\mu)\|_X}{\sqrt{\alpha_{LB}(\mu)}} \leq 1,$$

then we have an effective error bound for the relative error in the energy norm via

$$\frac{\|u(\mu) - u_N(\mu)\|_\mu}{\|u(\mu)\|_\mu} \leq \Delta_N^{en,rel}(\mu) \quad \text{and} \quad \eta_N^{en,rel}(\mu) := \frac{\Delta_N^{en,rel}(\mu)}{\|e(\mu)\|_\mu / \|u(\mu)\|_\mu} \leq 3 \cdot \sqrt{\frac{\gamma_{UB}(\mu)}{\alpha_{LB}(\mu)}}.$$

Exercise 3.3 (voting Exercise)

Prove Proposition 1.49: introducing the *dual residual* $r^{du}(v;\mu) := -l(v;\mu) - a(v, u_N^{du}(\mu);\mu)$ for $\mu \in \mathcal{P}$ and $v \in X$, we obtain the effective *dual error estimator* via

$$\left\|u^{du}(\mu) - u_N^{du}(\mu)\right\|_X \leq \Delta_N^{du}(\mu) := \frac{\left\|v_r^{du}(\mu)\right\|_X}{\alpha_{LB}(\mu)} \quad \text{and} \quad \eta_N^{du}(\mu) := \frac{\Delta_N^{du}(\mu)}{\left\|u^{du}(\mu) - u_N^{du}(\mu)\right\|_X} \leq \frac{\gamma_{UB}(\mu)}{\alpha_{LB}(\mu)},$$

as well as the improved output error bound

$$|s(\mu) - s'_N(\mu)| \leq \Delta'_s(\mu) := \frac{\left\|v_r^{du}(\mu)\right\|_X \|v_r(\mu)\|_X}{\alpha_{LB}(\mu)}.$$

Exercise 3.4 (programming Exercise)[5 Points]
Using the following commands, you can do a reduced simulation of the Thermal Block in `RBmatlab`, where for now the (unreasonable) Basis $\Phi = \{\mathbb{1}, e_1\}$ is used:

```
model = thermalblock_model_struct;
model_data = gen_model_data(model);
detailed_data = gen_detailed_data(model, model_data);
H = model_data.df_info.ndofs;
RB = [ones(H,1) eye(H,1)];
detailed_data.RB = RB;
reduced_data = gen_reduced_data(model, detailed_data);
model = set_mu(model, [1,0.1,1,0.1,1,0.1,1,0.1,1]);
rb_sim_data = rb_simulation(model, reduced_data);
rb_sim_data = rb_reconstruction(model,detailed_data,rb_sim_data);
plot_sim_data(model, model_data, rb_sim_data,[]);
```

Let the following parameters be given

$$\mu_1 = (1,1,1,1,1,1,1,1,1)^\intercal, \qquad \mu_2 = (1/10,1/10,1/10,1,1,1,1,1,1)^\intercal,$$
$$\mu_3 = (1/10,1,1,1/10,1,1,1/10,1,1)^\intercal, \quad \mu_4 = (1/2,1/2,1/2,1,1,1,1,1,1)^\intercal.$$

Modify the above commands, so that a reduced basis with snapshots for μ_1 and μ_2 is used.

(a) Typing `edit lin_stat_rb_simulation.m`, you can investigate how the reduced system matrix $\mathbf{A}_N(\mu)$ is generated. Compute for $i = 1,2,3,4$ the condition number of $\mathbf{A}_N(\mu_i)$ as well as $\gamma(\mu_i)/\alpha(\mu_i)$. Explain your results with respect to Proposition 1.28.

(b) Determine for $i = 1,2,3,4$

- the coefficient vector $\mathbf{u}_N(\mu_i)$,
- the error $\|u(\mu_i) - u_N(\mu_i)\|_X$,
- the output error $|s(\mu_i) - s_N(\mu_i)|$,

and interpret your results.

Tip: the function `fem_h10_norm` can be helpful for the $\|\cdot\|_X$-norm error calculation.

Explanation of terms

- For **written Exercises** hand in your elaboration/solution after one of the lectures or via Email to Dominik Garmatter until **25.11.2019**.
- For **programming Exercises** send your **commented** solution in `MATLAB`-Code via E-mail to Dominik Garmatter. Please, start the subject of your E-mail with **"MOR_201920_3:"**.
- For **voting Exercises** no written solution is requested. The solution for these exercises will be discussed/presented during the exercise course.
- All exercises of Exercise sheet 3 will be discussed during the exercise on 26.11.2019.