TECHNISCHE UNIVERSITÄT CHEMNITZ

Exercise sheet 2 – released on 29.10.2019

Exercise 2.1 (voting Exercise)

Prove Lemma 1.17: let l be a parametric continuous, parameter separable linear form and a a parametrically continuous, parameter separable bilinear form.

(a) If $\theta_q^l(\mu)$ or $\theta_q^a(\mu)$ are bounded for all $q = 1, \ldots, Q_l$ or $q = 1, \ldots Q_a$, then l or a are uniformly continuous w.r.t. μ.

(b) If there exists a constant $c > 0$ such that $\theta_q^a(\mu) \geq c$ for all $\mu \in \mathcal{P}$, $q = 1, \ldots, Q_a$, and if $a_q(v, v) \geq 0$ for all $v \in X$, $q = 1, \ldots Q_a$, and if $a(\cdot, \cdot; \mu)$ is coercive for at least one $\bar{\mu} \in \mathcal{P}$, then a is uniformly coercive w.r.t. μ.

(c) If $\theta_q^l(\mu)$ or $\theta_q^a(\mu)$ are Lipschitz-continuous for all $q = 1, \ldots, Q_l$ or $q = 1, \ldots, Q_a$, then l or a are Lipschitz-continuous w.r.t. μ.

Exercise 2.2 (written Exercise)[6 Points]

Investigate Example 1.18: for $\mathcal{P} := [\mu_{\min}, \mu_{\max}]^p \subset \mathbb{R}^p$ and $X := H^1_{\Gamma_D}(\Omega) := \{u \in H^1(\Omega) \mid u\,|_{\Gamma_D} = 0\}$ we have for $u,\ v \in X$ and $\mu \in \mathcal{P}$

$$a(u, v; \mu) := \int_\Omega \sigma(x; \mu) \nabla u(x; \mu) \cdot \nabla v(x) \,\mathrm{d}x \quad \text{and} \quad f(v; \mu) := \int_{\Gamma_{N,1}} v(x) \,\mathrm{d}x.$$

Note that X is equipped with the scalar product

$$\langle u, v \rangle_X := \int_\Omega \nabla u(x) \cdot \nabla v(x) \,\mathrm{d}x, \quad \text{and induced norm} \quad \|u\|_X := \sqrt{\langle u, u \rangle_X},$$

for $u,\ v \in X$.

(a) Show that $a(\cdot, \cdot; \mu)$ is a parameter-separable bilinear form and it is parametrically coercive with coercivity constant $\alpha(\mu) = \min_{i=1,\ldots,p} \mu_i$ and parametrically continuous with continuity constant $\gamma(\mu) = \max_{i=1,\ldots,p} \mu_i$.

(b) Conclude that a is uniformly coercive and uniformly continuous w.r.t. μ.

(c) Show that $f(\cdot; \mu)$ is a parameter-separable linear form and it is parametrically continuous. Conclude that it is also uniformly continuous w.r.t. μ.

Tip: use without proof (corollary of the trace theorem) that there exists a constant $C > 0$ so that $\|v\,|_{\partial\Omega}\|_{L^2(\partial\Omega)} \leq C \|v\|_X$ for all $v \in X$.

Exercise 2.3 (voting Exercise)

Show that the Thermal Block model from Example 1.18 for $B_1 = 1$ and $B_2 = 2$ has a solution manifold, which is contained in a 2-dimensional linear subspace $X_N \subset H^1_{\Gamma_D}(\Omega)$. You can make the following Ansatz:

- Derive coefficient functions $c_1(\mu),\ c_2(\mu)$ such that

$$u(x_1, x_2; \mu) := \begin{cases} c_1(\mu)x_2, & 0 \leq x_2 \leq \frac{1}{2}, \\ c_1(\mu)x_2 + c_2(\mu)(x_2 - \frac{1}{2}), & \frac{1}{2} \leq x_2 \leq 1, \end{cases}$$

  is a solution of $b(u, v; \mu) = f(v)$ for all $v \in H^1_{\Gamma_D}(\Omega)$.

  Tip: use without proof that it is sufficient to show that u solves $b(u, v; \mu) = f(v)$ for all

$$v \in C_0^\infty(\overline{\Omega} \setminus \Gamma_D) := \{v \in C^\infty(\overline{\Omega} \setminus \Gamma_D) \mid \operatorname{supp}(v) \text{ is compact and } \operatorname{supp}(v) \subset \overline{\Omega} \setminus \Gamma_D\},$$

  as this space is dense in $H^1_{\Gamma_D}(\Omega)$.

- Conclude that the solution manifold is actually 2-dimensional.

Exercise 2.4 (programming Exercise)[6 Points]
Install the `MATLAB` package `RBmatlab`, which will be used for future programming exercises. In order to do so, go to the homepage to the point **RBmatlab** in the section **Material**, where you find and download the `RBmatlab.zip` as well as extract it to a folder, e.g., `~/RBmatlab`. Now, create a `startup.m` file with the following content

```
setenv('RBMATLABHOME','~/RBmatlab');
setenv('RBMATLABTEMP','~/tmp/matlab');
addpath(getenv('RBMATLABHOME'));
startup_rbmatlab
```

and save this `startup.m` file in your home-folder (Linux) or in the folder `~\Documents\MATLAB\` if you are using Windows. Next, start `MATLAB` and verify the correct installation of `RBmatlab` by running `demo_detailed_gui`.

(a) Do a finite element (FEM) simulation of the 3×3 Thermal Block (Example 1.18) and visualize the result using the following commands:

```
params.B1 = 3;
params.B2 = 3;
params.numintervals_per_block = 5;
model = thermalblock_model_struct(params);
model = set_mu(model,[1,1,1,1,1,1,1,1,1]);
model_data = gen_model_data(model);
sim_data = detailed_simulation(model, model_data);
plot_sim_data(model, model_data,sim_data,[]);
```

This simulation uses a FEM-grid with 5 intervals per block and thus 15 intervals per edge of Ω.

(b) A field of the structure `sim_data` contains the value of the output functional, which was the average temperature over $\Gamma_{N,1}$. Determine the value of the output functional for the following parameters

$$\mu_1 := (1, 1, 1, 1, 1, 1, 1, 1, 1)^\intercal, \qquad \mu_2 := (1, 0.1, 0.01, 1, 0.1, 0.01, 1, 0.1, 0.01)^\intercal.$$

(c) Typing `edit lin_stat_detailed_simulation.m`, you can investigate how the finite element matrix A is generated. Compute the size of A and the condition number of A for

$$\mu = (1, 1, 1, 1, 1, 1, 1, 1, 1)^\intercal \quad \text{and for} \quad n = 2, 4, 8, 16, 32,$$

where n specifies the number of intervals per block.

(d) Generate a Thermal Block with $B_1 = 1$, $B_2 = 10$ and 5 intervals per block. In `model.mu_ranges` you can find μ_{min} and μ_{max} the smallest and largest value of the parameter. Do a FEM simulation and visualize the result for each of the following parameters

$$\mu_1 = (\mu_{\text{min}}, \dots, \mu_{\text{min}})^\intercal \quad \text{and} \quad \mu_j = (\mu_{\text{min}}, \dots, \mu_{\text{min}}, \underbrace{\mu_{\text{max}}}_{j-\text{th position}}, \mu_{\text{min}}, \dots, \mu_{\text{min}}), \quad j = 2, \dots, 10.$$

Furthermore, store the FEM-coefficients of these simulations, which can be accessed via `sim_data.uh.dofs`, in a matrix and compute the rank of this matrix.

Explanation of terms

- For **written Exercises** hand in your elaboration/solution after one of the lectures or via Email to Dominik Garmatter until **11.11.2019**.
- For **programming Exercises** send your **commented** solution in `MATLAB`-Code via E-mail to Dominik Garmatter. Please, start the subject of your E-mail with **"MOR_201920_2:"**.
- For **voting Exercises** no written solution is requested. The solution for these exercises will be discussed/presented during the exercise course.
- All exercises of Exercise sheet 2 will be discussed during the exercise on 12.11.2019.