Technische Universität Chemnitz

Sonderforschungsbereich 393

Numerische Simulation auf massiv parallelen Rechnern

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Two Boundary Element Methods for the clamped plate

Preprint SFB393/00-13

Preprint-Reihe des Chemnitzer SFB 393

SFB393/00-13 **March 2000**

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Abstract

In this paper we retail the approximation of the clamped plate problem by means of two boundary element methods. In both cases, the variational formulation is given on product of Sobolev spaces and we avoid the orthogonality to polynomials of degree one. The use of biorthogonal wavelets on these spaces leads to a well-conditioned stiffness matrix and reduces strongly the complexity thanks to a compression procedure. The solution of the compressed Galerkin scheme converges to the exact solution at the same rate as for the classic Galerkin method.

1 Introduction

The clamped plate problem involves the biharmonic operator of order 4. The integral formulation of the problem has been studied by several authors [5, 7, 8, 9, 10, 18, 20] among others. The advantage of this method is to reduce the dimension of the problem but it shows some drawbacks. When writing down the integral formulation by "single layer potential", we obtain a coupled system of unknowns (q_1, q_2) and a 2×2 system of pseudodifferential operators. This system is strongly elliptic on the product $H^{-3/2}(\Gamma) \times H^{-1/2}(\Gamma)$ and self-adjoint but the associated bilinear form is only positive definite on a subspace of the previous one, namely the subspace

$$\left\{(q_1, q_2) \in H^{-3/2}(\Gamma) \times H^{-1/2}(\Gamma) \text{ s.t. } \langle q_1, P\rangle + \langle q_2, \partial_n P\rangle = 0, \forall P \in \mathbf{P}_1\right\},$$

when $\mathbf{P}_1$ is the space of polynomials of degree one. In order to approximate the exact solution (q_1, q_2), we have to construct an approximation space and a basis satisfying this orthogonality condition, which seems to be difficult from a practical point of view.

Therefore, one can ask if we can avoid this condition and work directly on the product space given above. Doing this, we can show that existence and uniqueness of the solution is not guaranteed and it appears that, for certain values of the capacity cap Γ, the problem formulated like this has no more unique solution. For instance, when the curve Γ is a circle of radius R, the value $R = e^{-1}$ leads to nonuniquely solvable system (see [5, 18] and [7] for other examples).

In the first part of the paper, we use some results of [7] where the idea is to reformulate the problem on the scaled curve $\rho\Gamma$. The method show that a maximum of 4 values of ρ, which can be computed, have to be avoid. For all other scaling factors ρ the variational formulation of the problem is coercive and consequently, existence and unicity hold. We also present another integral formulation due early to G.C. Hsiao and R. MacCamy in [20] and continued by M. Costabel, E.P. Stephan and W.L. Wendland in [10] for the case of a polygonal boundary Γ. In this method, we add three new constraint equations to the system which has therefore 5 unknowns. Existence and uniqueness of the solution is achieved for all curve Γ.

In both methods, we use a wavelet basis for the discretization in order to avoid two other important drawbacks of general boundary element methods. Generally, the stiffness matrix is ill-conditioned and full, since the involved operators are nonlocal. Biorthogonal wavelets seem to be an efficient tool to avoid these two drawbacks. Wavelets have been studied from a theoretical point of view in [6, 12, 16, 21] and they are used for solving general elliptic partial differential equations, as in the precursory work [1] and later by several other authors [13, 14, 25], most of the time for elliptic operators of order 2. The originality of the present work is to extend the domain of using wavelets to more general operators, for instance to operators of order 4 (see also [3]).

We now explain how the paper is organised. In section 2, we recall some notations and we give the integral formulation of the problem. We mention the main properties of the involved operators and explain how we can avoid the use of spaces orthogonal to traces of polynomials of degree 1. In the next section, the Galerkin method is defined and one obtains a first error estimate due to the coercivity of the variational formulation. Section 4 is devoted to the construction of a biorthogonal wavelet basis which characterizes some Sobolev spaces in term of wavelet coefficients. With these functions, we get a simple preconditioner of the stiffness matrix in section 5. Furthermore, we can get a sparse compressed matrix with $O(N)$ nonzero elements instead of $O(N^2)$, by compressing two times. We complete the method by giving an error estimate between the exact solution and its wavelet-Galerkin approximation. In section 7, we present another approach for the formulation of the clamped plate problem by adding three constraints to the system of equations. The new system leads to a coercive variational formulation of the problem and the use of wavelets substantially reduces the complexity and improves the conditioning.

2 Integral formulation

Let $\Omega \subset \mathbf{R}^2$ be a bounded domain with a smooth boundary $\Gamma = \partial\Omega$ (C^2 for instance). We note $\Omega' = (\overline{\Omega})^c$ the open complementary of Ω in $\mathbf{R}^2$ and n the unitary outer normal vector on Γ.

For a more general domain $\Omega \subset \mathbf{R}^n$ and for a positive real $s = m + \sigma$, with $m \in \mathbf{N}$ and $\sigma \in (0,1)$, the function u belongs to the Sobolev space $H^s(\Omega)$ if and only if $\|u\|_{H^s(\Omega)} < \infty$ with

$$\|u\|_{H^s(\Omega)} = \left\{ \|u\|^2_{H^m(\Omega)} + |u|^2_{H^s(\Omega)} \right\}^{1/2}, \qquad (1)$$

with the semi-norm defined, for $|\alpha| = m$, by

$$|u|^2_{H^s(\Omega)} = \sum_{|\alpha|=m} \int\int_{\Omega^2} \frac{|D^\alpha u(x) - D^\alpha u(y)|^2}{|x-y|^{n+2\sigma}} dxdy. \tag{2}$$

We will note $\mathbf{P}_l$ the space of polynomials of degree less than or equal to l.

Because of the smoothness of the boundary Γ, the trace operators

$$\gamma_0 : u \longrightarrow \gamma_0 u := u_{|\Gamma}, \tag{3}$$

$$\gamma_1 : u \longrightarrow \gamma_1 u := \partial_n u_{|\Gamma}, \tag{4}$$

defined from $\mathcal{D}(\overline{\Omega})$ onto $\mathcal{C}^0(\Gamma)$ admit continuous extensions

$$(\gamma_0, \gamma_1) : H^s(\Omega) \longrightarrow H^{s-1/2}(\Gamma) \times H^{s-3/2}(\Gamma), \tag{5}$$

for $s > 3/2$.

In order to write the integral formulation of the problem, we introduce the weighted Sobolev spaces

$$L^2_1(\Omega') = \left\{u \in \mathcal{D}'(\Omega') | (1+r^2)^{-1}(\log(2+r^2))^{-1} u \in L^2(\Omega')\right\}, \tag{6}$$

$$W^2(\Omega') = \left\{u \in L^2_1(\Omega') | (1+r^2)^{-1/2}(\log(2+r^2))^{-1} \partial_{x_i} u \in L^2(\Omega'),\right. \tag{7}$$

$$\left. \partial_{x_i}\partial_{x_j} u \in L^2(\Omega'), \forall i,j = 1,2\right\}. \tag{8}$$

If we pose

$$\mathcal{B} := H^{3/2}(\Gamma)_{/\mathbf{P}_1} \times H^{1/2}(\Gamma)_{/\mathbf{C}}, \tag{9}$$

$\mathcal{B}$ is the space of the couples (g_0, g_1) such that there exists a function $u \in W^2(\mathbf{R}^2)/\mathbf{P}_1$ satisfying the boundary conditions $(\gamma_0 u, \gamma_1 u) = (g_0, g_1)$. We will note $\mathcal{B}'$ the dual space of $\mathcal{B}$.

The clamped plate problem reads as follow : find u solution of

$$\left\{ \begin{array}{rcll} \Delta^2 u &=& 0, & \text{in } \Omega, \\ u &=& g_0, & \text{on } \Gamma, \\ \partial_n u &=& g_1, & \text{on } \Gamma. \end{array} \right. \tag{10}$$

For $s \in \mathbf{R}$, and given data $g_0 \in H^{s+\frac{3}{2}}(\Gamma)$, $g_1 \in H^{s+\frac{1}{2}}(\Gamma)$, there exists a unique solution $u \in H^{s+2}(\Gamma)$ of problem (10).

We recall the fundamental solution of the biharmonic operator in $\mathbf{R}^2$:

$$G(x,y) = -\frac{1}{8\pi}|x-y|^2 \log|x-y|. \tag{11}$$

We introduce the following integral operators :

$$Au(x) \;:=\; -\int_\Gamma G(x,y)u(y)ds_y, \tag{12}$$

$$Bu(x) \;:=\; -\int_\Gamma \partial_{n_y} G(x,y)u(y)ds_y, \tag{13}$$

$$B'u(x) \;:=\; -\int_\Gamma \partial_{n_x} G(x,y)u(y)ds_y, \tag{14}$$

$$Cu(x) \;:=\; -\int_\Gamma \partial_{n_x}\partial_{n_y} G(x,y)u(y)ds_y, \tag{15}$$

for $x \in \Omega \cup \Omega'$. The solution of problem (10) admits the following single layer representation

$$u(x) = -\left\{Aq_1(x) + Bq_2(x)\right\}, \quad x \in \Omega \cup \Omega', \tag{16}$$

where (q_1, q_2) is a solution of the system

$$\left\{ \begin{array}{l} Aq_1(x) + Bq_2(x) = -g_0(x), \\ B'q_1(x) + Cq_2(x) = -g_1(x). \end{array} \right. \tag{17}$$

The above system of two equations suggests to introduce the operator

$$\mathcal{A} = \begin{pmatrix} A & B \\ B' & C \end{pmatrix}. \tag{18}$$

We recall the main properties of $\mathcal{A}$ (see [11]).

Lemma 2.1 *If the boundary Γ of the domain is smooth, the operator $\mathcal{A}$ is a strongly elliptic selfadjoint matrix of pseudodifferential operators of orders* $\begin{pmatrix} -3 & -2 \\ -2 & -1 \end{pmatrix}$.

Moreover, the system (17) defines a bounded positive definite bilinear form on the subspace of codimension 3 :

$$\mathcal{B}' := \left(H^{3/2}(\Gamma)_{/\mathbf{P}_1} \times H^{1/2}(\Gamma)_{/\mathbf{C}}\right)', \tag{19}$$

where $(\cdot)'$ denotes the duality. More precisely, the space $\mathcal{B}'$ is defined by

$$\mathcal{B}' = \left\{(q_1, q_2) \in H^{-3/2}(\Gamma) \times H^{-1/2}(\Gamma) \ s.t.\ \langle q_1, P\rangle + \langle q_2, \partial_n P\rangle = 0, \forall P \in \mathbf{P}_1\right\}. \tag{20}$$

We present hereafter some methods and results for which the bilinear form is still positive definite on the space $H^{-3/2}(\Gamma) \times H^{-1/2}(\Gamma)$, i.e. without the additional orthogonal condition.

We mention first a modification of the initial system, introduced by Fuglede [18] where only the second equation in (17) is modified and replaced by

$$-\frac{1}{2\pi}\int_\Gamma \left\{\log|x-y|q_1(y) - \partial_{n_y}\log|x-y|q_2(y)\right\} ds_y - \frac{1}{2}q_2(y) = 0.$$

For this modified system of equations, existence and unicity of the solution is proved in [18] for cap $\Gamma \notin \{1, \frac{1}{e}\}$, when cap is the capacity of a given curve. For instance when Γ is a circle of radius R, existence and uniqueness hold for $R \neq \frac{1}{e}$.

Coming back to the clamped plate problem, in [7] the authors study the solvability of the initial system (17), without any additional equation. The main result reads as follows: for any curve Γ, the problem (17) considered on the scaled curve

$$\rho\Gamma := \{\rho x \in \mathbf{R}^2 \text{ s.t. } x \in \Gamma\} \tag{21}$$

is uniquely solvable if and only if the scale factor $\rho > 0$ satisfies $\rho \notin S_\Gamma$, where card $S_\Gamma \in \{1, \dots, 4\}$. Let us remark that there is no particular assumption on the regularity of Γ which can be smooth or polygonal. In the latest case, we have to define carefully the Sobolev spaces $H^{-3/2}(\Gamma)$, $H^{-1/2}(\Gamma)$, and when one writes the trace theorem on $\partial\Omega$, some compatibility conditions appear at the corners (see [19]).

We recall the method used by M. Costabel and M. Dauge in [7].

The main idea is to reformulate the problem (17) in new spaces. The new formulation is equivalent to the initial one when the domain is smooth, no regularity assumption on Γ is needed (except for the polygonal case as explained before) and the problem becomes scalar. We introduce the following Sobolev spaces. Let us respectively define

$$\tilde{H}^2(\mathbf{R}^2 \setminus \Gamma) := \overline{\mathcal{C}_0^\infty(\mathbf{R}^2 \setminus \Gamma)}^{H^2(\mathbf{R}^2)}, \tag{22}$$

$$H^2_\gamma(\Gamma) := H^2(\mathbf{R}^2)_{/\tilde{H}^2(\mathbf{R}^2\setminus\Gamma)}. \tag{23}$$

We pose

$$\gamma : H^2(\mathbf{R}^2) \longrightarrow H^2_\gamma(\Gamma), \tag{24}$$

and

$$H^{-2}_\Gamma := \{q \in H^{-2}(\mathbf{R}^2) \text{ s.t. supp } q \subset \Gamma\}. \tag{25}$$

The above spaces have the following properties (see Lemma 4.1 in [7]).

Lemma 2.2 *For all boundary Γ, $H^{-2}_\Gamma = (H^2_\gamma(\Gamma))'$.*

If Γ is smooth, we have two isomorphisms :

$$H^2_\gamma(\Gamma) \sim H^{3/2}(\Gamma) \times H^{1/2}(\Gamma), \tag{26}$$

$$H^{-2}_\Gamma \sim H^{-3/2}(\Gamma) \times H^{-1/2}(\Gamma). \tag{27}$$

The latest one is defined, for a given $q \in H^2_\gamma(\Gamma)$ with $(q_1, q_2) \in H^{-3/2}(\Gamma) \times H^{-1/2}(\Gamma)$, we write

$$\langle q, \chi\rangle = \int_\Gamma (q_1\chi + q_2\partial_n\chi)ds, \quad \forall\chi \in \mathcal{C}_0^\infty(\mathbf{R}^2).$$

Therefore, the operator $\mathcal{A}$ defined by (18) acts also on H_Γ^{-2} :

$$\mathcal{A} : H_\Gamma^{-2} \longrightarrow H^2_\gamma(\Gamma).$$

We pose, as the notations in [7], $A = \mathcal{A}$, $X = H_\Gamma^{-2}$, $X' = H^2_\gamma(\Gamma)$, $\mathcal{P} = \text{span}\ \{p_0, p_1, p_2, p_3\}$ with $(p_0, p_1, p_2, p_3) = \gamma(1, x_1, x_2, |x|^2)$, γ beeing defined by (24), and

$$X_0 = \{f \in X : \langle f, p_i\rangle = 0, \forall i \in \{0, \dots, 3\}\}.$$

As explained before, operator $\mathcal{A}$ is positive on X_0. We therefore define the modified system of equations

$$\left\{ \begin{array}{lcl} \mathcal{A}q & = & \sum_{i=0}^3 \omega_i p_i, \\ \langle q, p_i\rangle & = & \xi_i, \qquad i = 0, \dots, 3, \end{array} \right. \tag{28}$$

which has, for all $\xi \in \mathbf{R}^4$, a unique solution $(q, \omega) \in H_\Gamma^{-2} \times \mathbf{R}^4$ and we introduce the linear application $\xi \longrightarrow \omega := B_\Gamma \xi$.

The compact set Γ has to verify the additional assumption :

($\mathcal{P}$) The traces of polynomials $1, x_1, x_2, |x|^2$ in $H^2_\gamma(\Gamma)$ are linearly independent.

The previous hypothesis is allways satisfied when the boundary Γ of the domain is a curve, for instance. Assuming now that the assumption ($\mathcal{P}$) is satisfied, we obtain a new expression of operator B_Γ under the scale transformation $\Gamma \longrightarrow \rho\Gamma$.

Lemma 2.3 *Let $\rho > 0$. Under the scale transformation (21), the operator B_Γ defined above becomes*

$$B_{\rho\Gamma} = D_\rho \left(B_\Gamma + \frac{\log \rho}{8\pi} C \right) D_\rho, \tag{29}$$

with

$$C = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad D_\rho = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \rho^{-1} \end{pmatrix}. \tag{30}$$

Therefore, the followowing theorem explains for which values of ρ the system (17) can be solved on the space H_Γ^{-2}.

Theorem 2.4 *If Γ is a compact set satisfying assumption ($\mathcal{P}$), the operator*

$$\mathcal{A}_\rho : H_{\rho\Gamma}^{-2} \longrightarrow H^2_\gamma(\rho\Gamma) \tag{31}$$

is not an isomorphism if and only if

$$\rho = e^{-8\pi\lambda}, \tag{32}$$

where λ is the eigenvalue of the matrix $C^{-1}B_\Gamma$.

In the rest of the paper, we assume that

$$\rho \notin S_\Gamma := \{e^{-8\pi\lambda} \; ; \; \lambda \in \text{Sp } (C^{-1}B_\Gamma)\}, \tag{33}$$

and we will write for short Γ instead of $\rho\Gamma$, with $\rho \notin S_\Gamma$.

3 Galerkin method

We begin to describe the variational formulation of the problem (10).

Let us define the bilinear form a by

$$a(q, q') = \langle \mathcal{A}q, q' \rangle_{H^2_\gamma(\Gamma) \times H_\Gamma^{-2}}, \quad \forall q' \in H_\Gamma^{-2}. \tag{34}$$

We are seeking for the unknown

$$q = (q_1, q_2) = \left(- \left[\frac{\partial(\Delta u)}{\partial n} \right], [\Delta u] \right) \in H_\Gamma^{-2}, \tag{35}$$

where $[\cdot]$ denotes the jump through the boundary Γ.

An explicit expression of the bilinear form a is given by

$$\begin{aligned} a(q, q') &= \int_{\Gamma\times\Gamma} q_1(x) q_1'(y) E(|x-y|) ds_x ds_y \\ &+ \int_{\Gamma\times\Gamma} q_2(x) q_1'(y) \partial_{n_x} E(|x-y|) ds_x ds_y \\ &+ \int_{\Gamma\times\Gamma} q_1(x) q_2'(y) \partial_{n_y} E(|x-y|) ds_x ds_y \\ &+ \int_{\Gamma\times\Gamma} q_2(x) q_2'(y) \partial_{n_x} \partial_{n_y} E(|x-y|) ds_x ds_y. \end{aligned} \tag{36}$$

Therefore, the variational formulation of the problem consists in finding $q \in H_\Gamma^{-2}$ solution of

$$a(q, q') = \langle (g_0, g_1), q' \rangle, \quad \forall q' \in H_\Gamma^{-2}. \tag{37}$$

In view of the previous section, we immediatly get

Proposition 3.1 *For all $\rho \notin S_\Gamma$, the bilinear form a is symmetric, continuous and coercive on H_Γ^{-2}.*

Given a positive integer J, we define the set of grid points $\{x_k\}_{k=0}^{2^J-1}$ on the boundary Γ and we note s_k the curvilinear abscissae of x_k on Γ. If c_Γ is the length of the curve Γ, we pose :

$$|s_{k+1} - s_k| = \frac{c_\Gamma}{2^{-J}}, \forall k = 0, \dots, 2^J - 1.$$

The approximation space S_J of H_Γ^{-2} is given by

$$S_J = \{(f,g) \in S_J^L \times S_J^M\}, \tag{38}$$

where

$$S_J^L = \{f \,:\, f_{|[s_k,s_{k+1}]} \in \mathbf{P}_L, k = 0,\ldots,2^J-1\}, \tag{39}$$

with

$$L \geq s - \frac{5}{2}, \quad M \geq s - \frac{3}{2}. \tag{40}$$

Using the Galerkin method, we want to find $q_J \in S_J$ solution of

$$a(q_J, q'_J) = \langle (g_0, g_1), q'_J \rangle, \quad \forall q'_J \in S_J. \tag{41}$$

Due to the coercivity of the bilinear form, we have the error estimate :

Theorem 3.2 *Let $q = (q_1, q_2) \in H^{s-3/2}(\Gamma) \times H^{s-1/2}(\Gamma)$ be the exact solution of problem (17) with data $(g_0, g_1) \in H^{s+3/2}(\Gamma) \times H^{s+1/2}(\Gamma)$ and suppose that (40) is satisfied. Therefore, if $\rho \notin S_\Gamma$ and if q_J is the Galerkin approximation of q, we have*

$$\|q - q_J\|_{H_\Gamma^{-2}(\Gamma)} \lesssim 2^{-sJ} \left(\|g_0\|^2_{H^{s+3/2}(\Gamma)} + \|g_1\|^2_{H^{s+1/2}(\Gamma)} \right)^{1/2}. \tag{42}$$

Proof: By Céa's Lemma, we may write

$$\|q - q_J\|_{H_\Gamma^{-2}(\Gamma)} \lesssim \|q - Q_J q\|_{H_\Gamma^{-2}(\Gamma)},$$

where Q_J is the $L^2(\Gamma)^2$-projection on S_J. Because of the regularity of the data, we have $q_1 \in H^{s-3/2}$ and $q_2 \in H^{s-1/2}$. Now, if the approximation space admits a spline basis, it is well-known from approximation theory that

$$\|q - Q_J q\|^2_{H^{-2}(\Gamma)} \lesssim 2^{2J(-3/2-(s-3/2))} \|g_0\|^2_{H^{s+3/2}(\Gamma)} + 2^{2J(-1/2-(s-1/2))} \|g_1\|^2_{H^{s+1/2}(\Gamma)}.$$

■

Instead of using a classical spline basis of the approximation space S_J, we construct in the next section a biorthogonal wavelet basis which characterizes spaces $H^s(\Gamma) \times H^{s+1}(\Gamma)$. This new basis will lead to a simple preconditioning and a compression of the stiffness matrix.

4 Biorthogonal wavelet basis

We recall in this section the main steps for the construction of a biorthogonal wavelet basis. We refer to [12, 13, 14, 25] for further details.

Given a Hilbert space H (for instance $H = L^2(\Gamma)$ or $H = H^s(\Gamma)$), we suppose that we have a sequence of nested closed subspaces S_j of H, whose union is dense in H :

$$S_0 \subset S_1 \subset \cdots \subset H,$$
$$\text{clos}_H \left(\bigcup_{j=0}^{\infty} S_j \right) = H.$$

The spaces S_j have the form

$$S_j = S(\Phi_j) = \text{clos}_H(Span(\Phi_j)), \quad \Phi_j = \{\varphi_{j,k} : k \in \Delta_j\},$$

with Δ_j a countable set of indices and Φ_j are stable bases.

From the identity

$$S(\Phi_{j+1}) = S(\Phi_j) \bigoplus S(\Psi_j),$$

we introduce the set $W_j = S(\Psi_j)$, the complement of $S(\Phi_j)$ in $S(\Phi_{j+1})$. The set $\Psi_j = \{\psi_{j,k} : k \in \nabla_j\}$ is the collection of the successive translates of the wavelet ψ_j, at a fixed level j.

For a function ψ, on $\mathbf{R}^n$ to fix the ideas, we will note

$$\Psi = \{\psi_\lambda : \lambda \in \nabla\},$$

where the indices $\lambda \in \nabla$ encode the level of resolution, which will be denoted by $|\lambda|$ (or j as in the previous notation), the location of the function (k) and sometimes the type of wavelet which is used (e). ψ_λ can be written as

$$\psi_\lambda = 2^{jn/2} \psi_e(2^j \cdot -k). \tag{43}$$

Sometimes it is useful to use the condensed notation (43) instead of the original one $\psi_{j,k}(x) = 2^{jn/2}\psi(2^j x - k)$.

Now suppose that are given two sets of functions :

$$\begin{aligned} \Psi &= \{\psi_{j,k} : (j,k) \in \nabla\}, \\ \tilde{\Psi} &= \{\tilde{\psi}_{j,k} : (j,k) \in \nabla\}, \end{aligned}$$

where $\nabla = \{(j,k) : k \in \nabla_j, j = -1, 0, 1, 2, \ldots\}$ such that

$$\langle \psi_{j,k}, \tilde{\psi}_{j',k'} \rangle = \delta_{(j,k),(j',k')}, \quad (j,k), (j',k') \in \nabla, \tag{44}$$

where $\langle \cdot, \cdot \rangle$ is the scalar product in H.

Let $\kappa : \Gamma \longrightarrow [0,1]$ be a smooth parametrisation of the boundary Γ. We also introduce, for $\varphi \in \mathcal{C}_0^\infty(\Gamma)$, the function $\tilde{\kappa}$ defined by

$$\tilde{\kappa} \circ \varphi(x) = \varphi\left(\kappa^{-1}(x)\right). \tag{45}$$

The wavelets have the two following fundamental properties, which are essential when writing down the Galerkin method and the compression. First of all, wavelets ψ and their dual $\tilde{\psi}$ have a compact support. In the following, we will note

$$\Omega_{j,k} := \text{supp } \psi_{j,k}, \tag{46}$$

the support of the wavelet $\psi_{j,k}$. By the same way and as needed when writing the compression, we also introduce

$$\Omega^s_{j,k} := \text{Sing supp } \psi_{j,k}, \tag{47}$$

for the singular support of $\psi_{j,k}$.

On the other hand, if κ is defined as above, the wavelets satisfy the moment property

$$\int_{\mathbf{R}} x^\alpha \psi_{j,k}(\kappa^{-1}(x))dx = 0, \quad |\alpha| \leq \tilde{d}, \quad k \in \nabla_j. \tag{48}$$

That is, ψ has $\tilde{d}+1$ vanishing moments. Thanks to these assumptions, we will be able to compress the stiffness matrix in section 5. Let us define the following projectors : for $u \in L^2(\mathbf{R})$, we note Q_j the projection on S_j and pose

$$\begin{aligned} (Q_{j+1} - Q_j)\, u &= \sum_{k\in\nabla_j} \langle u, \tilde{\psi}_{j,k}\rangle \psi_{j,k}, & (49)\\ \left(\tilde{Q}_{j+1} - \tilde{Q}_j\right) u &= \sum_{k\in\nabla_j} \langle u, \psi_{j,k}\rangle \tilde{\psi}_{j,k}. & (50)\end{aligned}$$

With the above biorthogonal system, every $v \in H$ has a unique expansion in these bases of the following form :

$$v = \sum_{(j,k)\in\nabla} \langle v, \tilde{\psi}_{j,k}\rangle \psi_{j,k} = \sum_{(j,k)\in\nabla} \langle v, \psi_{j,k}\rangle \tilde{\psi}_{j,k} \tag{51}$$

such that the systems are stable in the sense that

$$\|v\|_H^2 \sim \sum_{(j,k)\in\nabla} |\langle v, \tilde{\psi}_{j,k}\rangle|^2 = \sum_{(j,k)\in\nabla} |\langle v, \psi_{j,k}\rangle|^2. \tag{52}$$

Such a dual system is a good candidate for the characterization of the usual Sobolev spaces on $\mathbf{R}^n$ by means of the wavelet coefficients if the Bernstein and Jackson estimates hold. We recall the following general result ([12, 13]).

Theorem 4.1 *Let us assume that the Jackson estimate holds, namely*

$$\|v - Q_j v\|_\tau \lesssim 2^{j(\tau-t)}\|v\|_t, v \in H^t(\Gamma), \tag{53}$$

for $-\tilde{d}-1 < \tau < \gamma, \tau \leq t, -\tilde{\gamma} < t \leq d+1$*, with a similar inequality for* $(v - \tilde{Q}_j v)$*, by interchanging* d *and* $\tilde{d}$*,* γ *and* $\tilde{\gamma}$*.*

Moreover, if we have the following "inverse" property

$$\|v_j\|_t \lesssim 2^{j(t-\tau)} \|v_j\|_\tau, v_j \in S_j, \tag{54}$$

if $-\infty < \tau \leq t < \tilde{\gamma}$ *; and*

$$\|v_j\|_t \lesssim 2^{j(t-\tau)} \|v_j\|_\tau, v_j \in \tilde{S}_j, \tag{55}$$

for $-\infty < \tau \leq t < \gamma$*, the next eqivalences hold :*

$$\|v\|_t \sim \sum_{k\in\Delta_{j_0}} |\langle v, \varphi_{j_0,k}\rangle|^2 + \sum_{j=j_0}^{\infty} \sum_{k\in\nabla_j} 2^{2jt} |\langle v, \tilde{\psi}_{j,k}\rangle|^2, \tag{56}$$

$$\|v\|_t \sim \sum_{k\in\Delta_{j_0}} |\langle v, \varphi_{j_0,k}\rangle|^2 + \sum_{j=j_0}^{\infty} \sum_{k\in\nabla_j} 2^{2jt} |\langle v, \psi_{j,k}\rangle|^2, \tag{57}$$

for $-\tilde{\gamma} < t < \gamma, -\gamma < t < \tilde{\gamma}$*.*

Remark 4.2 *The choice* $d = \tilde{d} = 2$ *(i.e. the use of piecewise linear elements) allows to characterize* $H^s(\mathbf{R})$ *for* $s \in [-\frac{1}{2}, \frac{3}{2})$ *and in this case,* $\gamma = \frac{3}{2}$*.*

Now we apply the previous general construction to our concrete problem. Suppose that, with the above construction, we have a set of wavelets $\{\psi_{d_1;j,k}\}$, exact of order d_1, which characterize $H^{s_1}(\mathbf{R})$ for $s_1 \in (-\tilde{\gamma}_1, \gamma_1)$, with the assumption $\tilde{\gamma}_1 > -\frac{r}{2}$, $\gamma_1 > -\frac{r}{2}$. Because $r = -3$, we immediatly get that the value $s_1 = -\frac{3}{2}$ belongs to the interval $(-\tilde{\gamma}_1, \gamma_1)$.

By the same way, we construct a second set of wavelets $\{\psi_{d_2;j,k}\}$ which characterizes $H^{s_2}(\mathbf{R})$ for $s_2 \in (-\tilde{\gamma}_2, \gamma_2)$. The case $s_2 = -\frac{1}{2}$ is again included. Consequently, using a smooth parametrisation κ of the boundary $\partial\Omega$ and for $j \geq j_0 > 0$, we define the following wavelets in $L^2(\Gamma^2)$:

$$\psi^1_{j,k} := \begin{pmatrix} \psi_{d_1;j,k} \\ 0 \end{pmatrix}, \quad k \in \nabla^1_j \tag{58}$$

$$\psi^2_{j,k} := \begin{pmatrix} 0 \\ \psi_{d_2;j,k} \end{pmatrix}, \quad k \in \nabla^2_j \tag{59}$$

(resp. $\tilde{\psi}^1_{j,k}$, $\tilde{\psi}^2_{j,k}$ for the dual system). The integer j_0 is chosen sufficiently large such that supp $\psi_{d_i;j_0,0} \subset [0,1]$ for $i = 1, 2$. With the above wavelets we get the theorem

Theorem 4.3 *For* $\sigma \in \mathbf{R}_+$ *and for all* $s \in (-\frac{3}{2}, -\frac{3}{2}+\sigma)$*, we choose* (γ_1, γ_2) *such that*

$$\gamma_1, \gamma_2 > -\frac{1}{2} + \sigma. \tag{60}$$

For all $q = (q_1, q_2) \in H^s(\Gamma) \times H^{s+1}(\Gamma)$ *such that*

$$q = \sum_{k\in\Delta_{j_0}} \langle q, \varphi_{j_0,k}\rangle \varphi_{j_0,k} + \sum_{j=j_0}^{\infty} \left[\sum_{k\in\nabla_j^1} \langle q, \tilde{\psi}_{j,k}^1\rangle \psi_{j,k}^1 + \sum_{k\in\nabla_j^2} \langle q, \tilde{\psi}_{j,k}^2\rangle \psi_{j,k}^2 \right], \tag{61}$$

the next equivalence holds

$$\begin{aligned} \|q\|^2_{H^s(\Gamma)\times H^{s+1}(\Gamma)} &\sim \sum_{k\in\Delta_{j_0}} |\langle q, \varphi_{j_0,k}\rangle|^2 \\ &+ \sum_{j=j_0}^{\infty} \left\{ \sum_{k\in\nabla_j^1} 2^{2sj} |\langle q, \tilde{\psi}_{j,k}^1\rangle|^2 + \sum_{k\in\nabla_j^2} 2^{2(s+1)j} |\langle q, \tilde{\psi}_{j,k}^2\rangle|^2 \right\}. \end{aligned} \tag{62}$$

Proof: We start to write the definition of the norm of q :

$$\|q\|^2_{H^s(\Gamma)\times H^{s+1}(\Gamma)} := \|q_1\|^2_{H^s(\Gamma)} + \|q_2\|^2_{H^{s+1}(\Gamma)},$$

and we estimate separetly the two norms on the right hand side. In view of the properties of $\psi_{d_1;j,k}$ and with assumption (60), we have

$$\|q_1\|^2_{H^s(\Gamma)} \sim \sum_k |\langle q_1, \varphi^1_{j_0,k}\rangle|^2 + \sum_{j=j_0}^{\infty} \sum_{k\in\nabla_j^1} 2^{2sj} |\langle q_1, \tilde{\psi}_{d_1;j,k}\rangle|^2,$$

where

$$\varphi^1_{j_0,k} := \begin{pmatrix} \varphi_{j_0,k} \\ 0 \end{pmatrix}.$$

The conclusion holds because of the equality

$$\langle q_1, \tilde{\psi}_{d_1;j,k}\rangle = \langle q, \tilde{\psi}^1_{j,k}\rangle.$$

We estimate the second norm using the definition of $\psi_{d_2;j,k}$ and again assumption (60). Namely, we have for $\varphi^2_{j_0,k} = \begin{pmatrix} 0 \\ \varphi_{j_0,k} \end{pmatrix}$:

$$\begin{aligned} \|q_2\|^2_{H^{s+1}(\Gamma)} &\sim \sum_k |\langle q_2, \varphi^2_{j_0,k}\rangle|^2 + \sum_{j=j_0}^{\infty} \sum_{k\in\nabla_j^2} 2^{2(s+1)j} |\langle q_2, \tilde{\psi}_{d_2;j,k}\rangle|^2, \\ &\sim \sum_k |\langle q, \varphi_{j_0,k}\rangle|^2 + \sum_{j=j_0}^{\infty} \sum_{k\in\nabla_j^2} 2^{2(s+1)j} |\langle q, \tilde{\psi}^2_{j,k}\rangle|^2. \end{aligned}$$

■

As a consequence of the above Theorem, we immediatly get an error estimate for the Wavelet-Galerkin method.

Corollary 4.4 *If Q_i denotes the projection on V_i along W_i, with $i \geq j_0$, for any $(s,t) \in (-\frac{3}{2}, -\frac{3}{2}+\sigma)$, $s \leq t$, we have for all $q \in H^t(\Gamma) \times H^{t+1}(\Gamma)$:*

$$\|q - Q_i q\|_{H^s(\Gamma)\times H^{s+1}(\Gamma)} \lesssim 2^{i(s-t)} \|q\|_{H^t(\Gamma)\times H^{t+1}(\Gamma)}. \tag{63}$$

Proof: If we write shortly $q - Q_i q$ in the wavelet basis of $L^2(\Gamma^2)$ as

$$q - Q_i q = \sum_{j=i}^{\infty} \left[\sum_{k\in\nabla_j^1} c_{j,k}^1 \psi_{j,k}^1 + \sum_{k\in\nabla_j^2} c_{j,k}^2 \psi_{j,k}^2 \right],$$

and using Theorem 4.3, one has

$$\begin{aligned} \|q - Q_i q\|^2_{H^s(\Gamma)\times H^{s+1}(\Gamma)} &\sim \sum_{j=i}^{\infty} \left\{ \sum_{k\in\nabla_j^1} 2^{2sj} |c_{j,k}^1|^2 + \sum_{k\in\nabla_j^2} 2^{2(s+1)j} |c_{j,k}^2|^2 \right\} \\ &\lesssim 2^{2(s-t)i} \sum_{j=i}^{\infty} \left\{ \sum_{k\in\nabla_j^1} 2^{2tj} |c_{j,k}^1|^2 + \sum_{k\in\nabla_j^2} 2^{2(t+1)j} |c_{j,k}^2|^2 \right\} \\ &\lesssim 2^{2(s-t)i} \|q\|_{H^t(\Gamma)\times H^{t+1}(\Gamma)}, \end{aligned}$$

the last estimate is obtained by applying once more Theorem 4.3. ■

Remark 4.5 *If we apply the previous estimate with $t = s - \frac{3}{2}$, $s = -\frac{3}{2}$ and $i = J$, we find again the same estimate as in the classical Galerkin method, using a spline basis of S_J (see Theorem 3.2).*

From now on, we note shortly

$$\{\Psi_{j,k}\}_{k\in\nabla_j} := \{\psi_{j,k}^1\}_{k\in\nabla_j^1} \cup \{\psi_{j,k}^2\}_{k\in\nabla_j^2}, \tag{64}$$

as a wavelet basis of S_j and we pose

$$\{\Psi\} = \bigcup_{j,k\in\nabla} \{\Psi_{j,k}\}. \tag{65}$$

The functions $\Psi_{j,k}$ of the set $\{\Psi\}$ are exact of order d with $d \in \{d_1, d_2\}$ and they have $(\tilde{d}+1)$ vanishing moments for $\tilde{d} \in \{\tilde{d}_1, \tilde{d}_2\}$. For the sake of brevity, we write in the same way $\gamma \in \{\gamma_1, \gamma_2\}$, and define analogously $\tilde{\gamma}$.

5 Preconditioning and compression of the stiffness matrix

For $\nabla^J = \{(j,k) : j = -1, \ldots, J \ , \ k \in \nabla_j\}$, let

$$A_J = (\langle \mathcal{A}\Psi_{j,k}, \Psi_{j',k'} \rangle)_{(j,k),(j',k')\in\nabla^J}, = \left(A_{(j,k),(j',k')}\right)_{(j,k),(j',k')\in\nabla^J}, \tag{66}$$

be the stiffness matrix of $\mathcal{A}$ in the wavelet basis Ψ.

When the operator $\mathcal{A}$ is of order $r \neq 0$ and the Hilbert space $H = H^s(\Gamma)$ with $s \neq 0$, we have a simple diagonal preconditioner for the matrix A_J.

Proposition 5.1 *For $i \in \{1,2\}$, let $D^i_{s,j}$ be the diagonal matrix defined by*

$$\left(D^i_{s,j}\right)_{(j,k),(j',k')} = 2^{(s-i+1)j}\delta_{(j,k),(j',k')}, \quad (j,k),(j',k') \in \nabla^i_j, \tag{67}$$

and let us pose

$$D_{s,j} = \begin{pmatrix} D^1_{s,j} & 0 \\ 0 & D^2_{s,j} \end{pmatrix}. \tag{68}$$

If $\tilde{\gamma} > \frac{3}{2}$, we have :

$$cond\ (D_{3/2,J} A_J D_{3/2,J}) \leq C, \tag{69}$$

for a constant C independent of J.

Proof: By the definition of the matrices $D^i_{s,j}$ for $i = 1, 2$, the modified stiffness matrix has entries computed on the new wavelet basis functions $\overline{\Psi_{j,k}} = \{\overline{\psi^1_{j,k}}\} \cup \{\overline{\psi^2_{j,k}}\}$ with :

$$\overline{\psi^1_{j,k}} = 2^{3j/2}\psi^1_{j,k}, \tag{70}$$

$$\overline{\psi^2_{j,k}} = 2^{j/2}\psi^2_{j,k}. \tag{71}$$

Due to the Theorem of characterization 4.3 applied with $s = -3/2$ and because of the coercivity of the bilinear form a (see Proposition 3.1), the result follows immediately. ■

Using wavelets, we are also able to compress the stiffness matrix. The "compressed" matrix will only have $O(N)$ non-zero elements instead of $O(N^2)$ entries. One essential property we need is the decreasing of the kernel E of our operator, namely the bilaplacian here. This property exhibits a decay for the stiffness matrix coefficients, which is the starting point for the compression. These results are summarized in the next Lemma.

Lemma 5.2 *The following estimates hold :*

1. *The kernel K_{Δ^2} of the biharmonic operator satisfies :*

$$\left|\partial^\alpha_x \partial^\beta_y K_{\Delta^2}(x,y)\right| \lesssim [dist_\Gamma(x,y)]^{-(-2+|\alpha|+|\beta|)}, \tag{72}$$

when $-2 + |\alpha| + |\beta| > 0$.

The wavelelet coefficents of the stiffness matrix decrease in the following way :

2. *Suppose that $\tilde{d} > 0$ and $dist(\Omega_{j,k};\Omega_{j',k'}) > 0$, with $\Omega_{j,k}$ defined by (46). We have*

$$\left|A_{(j,k),(j',k')}\right| = \left|\langle \mathcal{A}\Psi_{(j,k)}, \Psi_{(j',k')}\rangle\right| \lesssim \frac{2^{-(j+j')(\tilde{d}+\frac{3}{2})}}{dist(\Omega_{j,k};\Omega_{j',k'})^{\tilde{d}}}. \tag{73}$$

3. Let $j' \leq j$, $0 \leq d < \tilde{d} - 3$ and $\gamma > -\frac{3}{2}$. We note $\Omega_{j',k'} = \cup_{\nu=1}^{n_\nu}\{\Sigma_\nu^{j'}\}$ where $s_\nu^{j'} := \Sigma_\nu^{j'-1} \cap \Sigma_\nu^{j'}$ are the points of $\Omega_{j',k'}$ on which the wavelet $\Psi_{j',k'}$ is not smooth. Suppose now that

$$\Psi_{j',k'}(\kappa^{-1})|\Sigma_\nu^{j'} \in \mathbf{P}_d. \tag{74}$$

If we assume that

$$\frac{1}{c}2^{-j'} \geq dist(\Omega_{j,k}, \Omega_{j',k'}^s) \geq c2^{-j}, \tag{75}$$

we have

$$|\langle \mathcal{A}\Psi_{j,k}, \Psi_{j',k'}\rangle| \lesssim 2^{-j(\tilde{d}+1/2)}2^{j'/2}dist(\Omega_{j,k}, \Omega_{j',k'}^s)^{2-\tilde{d}}. \tag{76}$$

Proof: By definition of the bilinear form a in (36), we have

$$a(\Psi_{j,k}; \Psi_{j',k'}) = \sum_{i=1}^{4} \int\int_{\Gamma^2} \Psi_{j,k}(x)\Psi_{j',k'}(y)k_i(x,y)ds_x ds_y,$$

with

$$k_1(x,y) = E(|x-y|) \quad , \quad k_2(x,y) = \partial_{n_x}E(|x-y|),$$
$$k_3(x,y) = \partial_{n_y}E(|x-y|) \quad , \quad k_4(x,y) = \partial_{n_x}\partial_{n_x}E(|x-y|).$$

Now we estimate the successive derivatives of each k_i by applying the Leibniz rule and we get immediatly the first assertion of the Lemma.

Because of the smoothness of Γ, we use the parametrization of the boundary κ defined in (45) such that

$$\begin{aligned} a(\Psi_{j,k}; \Psi_{j',k'}) &= \sum_i \int\int_{[0,1]^2} \Psi_{j,k}(\kappa^{-1}(x))\Psi_{j',k'}(\kappa^{-1}(y)) \\ &\times \; k_i(\kappa^{-1}(x), \kappa^{-1}(y))(\kappa^{-1})'(x)(\kappa^{-1})'(y)dxdy. \end{aligned}$$

From now on we write shortly $g_i(x,y) = k_i(\kappa^{-1}(x), \kappa^{-1}(y))(\kappa^{-1})'(x)(\kappa^{-1})'(y)$, for $i = 1,\ldots,4$. Therefore we may write

$$\langle \mathcal{A}\Psi_{j,k}, \Psi_{j',k'}\rangle = \sum_{i=1}^{4} \int_{\mathbf{R}^2} g_i(x,y)\Psi_{j,k}(\kappa^{-1}(y))\Psi_{j',k'}(\kappa^{-1}(x))dxdy. \tag{77}$$

We now develop $g_i(x,y)$ in a Taylor serie in two steps.

First of all, for $\kappa^{-1}(y) \in \Omega_{j,k}$, we develop the function $g_i(x,\cdot)$ as a function of x around the point $\kappa^{-1}(x_0) \in \Omega_{j',k'}$ and until order $\tilde{d}$, that is :

$$g_i(x,y) = \sum_{|\tilde{\alpha}| \leq \tilde{d}} c_{\tilde{\alpha}}^i(x_0,y)(x-x_0)^{\tilde{\alpha}} + R_{\tilde{d}+1}^i(x,x_0,y),$$

where $R^i_{\tilde d+1}$ is the integral rest of Lagrange of the Taylor expansion and is defined by

$$R^i_{\tilde d+1}(x,x_0,y)=\sum_{|\alpha|=\tilde d+1}\frac{(x-x_0)^\alpha(\tilde d+1)}{\alpha!}\int_0^1(1-t_1)^{\tilde d}\partial_x^\alpha g_i(x_0+t_1(x-x_0),y)dt_1.$$

Now if we introduce this Taylor expansion in the integral (77), due to the moment property of the wavelets, the first term of the Taylor development disappears and only $R^i_{\tilde d+1}$ has to be taken into account.

We do the same development for the function $y\longrightarrow R^i_{\tilde d+1}(x,x_0,y)$ for $\kappa^{-1}(x)\in\Omega_{j',k'}$ and for $\kappa^{-1}(y_0)\in\Omega_{j,k}$:

$$R^i_{\tilde d+1}(x,x_0,y)=\sum_{|\alpha|\le\tilde d}c^i_\alpha(x,x_0,y_0)(y-y_0)^\alpha+R^i_{\tilde d+1}(x,x_0,y,y_0),\tag{78}$$

where, by definition,

$$R^i_{\tilde d+1}(x,x_0,y,y_0)=\sum_{|\beta|=\tilde d+1}\frac{(y-y_0)^\beta(\tilde d+1)}{\beta!}\int_0^1(1-t_2)^{\tilde d}\partial_y^\beta R^i_{\tilde d+1}(x,x_0,y_0+t_2(y-y_0))dt_2.\tag{79}$$

If we replace in the integral, we obtain

$$\begin{aligned}|\langle\mathcal{A}\Psi_{j,k},\Psi_{j',k'}\rangle| &\lesssim \sum_{i=1}^4\left|\int_{\mathbf{R}^2}R^i_{\tilde d+1}(x,x_0,y,y_0)\Psi_{j,k}(\kappa^{-1}(y))\Psi_{j',k'}(\kappa^{-1}(x))dxdy\right|\\
&\lesssim \sum_{i=1}^4\sum_{|\alpha|,|\beta|=\tilde d+1}\int_{\mathbf{R}^2}|y-y_0|^{\tilde d+1}|x-x_0|^{\tilde d+1}|\Psi_{j,k}(\kappa^{-1}(y))\Psi_{j',k'}(\kappa^{-1}(x))|\\
&\times\left|\int_{[0,1]^2}(1-t_2)^{\tilde d}(1-t_1)^{\tilde d}\partial_y^\beta\partial_x^\alpha g_i(x_0+t_1(x-x_0),y_0+t_2(y-y_0))dt_1dt_2\right|dxdy,\\
&\lesssim \sum_{i=1}^4\sum_{|\alpha|,|\beta|}\int_{\mathbf{R}^2}|x-x_0|^{\tilde d+1}|y-y_0|^{\tilde d+1}\mathrm{Sup}_{\substack{\kappa^{-1}(y)\in\Omega_{j,k}\\ \kappa^{-1}(x)\in\Omega_{j',k'}}}|\partial_y^\beta\partial_x^\alpha g_i(x,y)|\\
&\times|\Psi_{j,k}(\kappa^{-1}(y))\Psi_{j',k'}(\kappa^{-1}(x))|dxdy.\end{aligned}$$

By the definition of k_i we have

$$\begin{aligned}|\langle\mathcal{A}\Psi_{j,k},\Psi_{j',k'}\rangle| &\lesssim \sum_{|\alpha|,|\beta|}\mathrm{dist}(\Omega_{j,k},\Omega_{j',k'})^{-2\tilde d}\\
&\times\int_{\mathbf{R}^2}|x-x_0|^{\tilde d+1}|y-y_0|^{\tilde d+1}|\Psi_{j,k}(\kappa^{-1}(y))\Psi_{j',k'}(\kappa^{-1}(x))|dxdy.\end{aligned}$$

We finish the proof by estimating the following integral using the definition of $\Psi_{j,k}$ and a change of variables in

$$\int_{\mathbf{R}}|x-x_0|^{\tilde d+1}|\Psi_{j',k'}(\kappa^{-1}(x))|dx\lesssim 2^{-j'(\tilde d+\frac{3}{2})}\int(u+k)^{\tilde d+1}|\Psi(\kappa^{-1}(u))|du,$$

and the last integral here above is uniformly bounded, independently of j and k because Ψ has a compact support.

The proof of the third assertion of the Lemma is established in three steps. We begin to estimate the following coefficients :

$$\begin{aligned} |\langle \mathcal{A}\Psi_{j,k'}, \varphi_{j,k}\rangle| &\lesssim \sum_i \sum_{|\alpha|=\tilde{d}+1} \sup_{\substack{\kappa^{-1}(x)\in\Omega_{j,k'} \\ y\in \mathrm{Supp}_{\varphi_{j,k}}}} |\partial_x^\alpha g_i(x,y)| \\ &\times \int_{\mathbf{R}^2} |(x-x_0)^\alpha||\Psi_{j,k'}(\kappa^{-1}(x))\varphi_{j,k}(\kappa^{-1}(y))|dxdy \\ &\lesssim \frac{2^{-j/2}2^{-j(\tilde{d}+3/2)}}{\mathrm{dist}(\Omega_{j,k'}, \mathrm{Supp}\varphi_{j,k})^{\tilde{d}-1}} \\ &\lesssim \frac{2^{-3j}}{[1+2^j \mathrm{dist}(\Omega_{j,k'}, \mathrm{Supp}\varphi_{j,k})^{\tilde{d}-1}}. \end{aligned}$$

Under the assumption (74), we have the moment property

$$\int_{\mathbf{R}} \Psi_{j',k'}(\kappa^{-1}(x))x^\alpha dx = 0, |\alpha| \le \tilde{d}.$$

Consequently, we can write, for $x \in \Sigma_\nu^{j'}$,

$$\Psi_{j',k'}(\kappa^{-1}(x)) = 2^{j'/2} \sum_{|\alpha|<d+1} c_\alpha [2^{j'}(x-x_\nu)]^\alpha.$$

If we note $\tau_\nu^{j'} = \kappa^{-1}\Sigma_\nu^{j'}$, and if $\tilde{\mathcal{A}}$ is defined as the restriction of $\mathcal{A}$ on $\Sigma_\nu^{j'}$, we get

$$\begin{aligned} |\langle \mathcal{A}\Psi_{j,k}, \Psi_{j',k'}\rangle| &= \left|\int_{\mathbf{R}} \left(\mathcal{A}\Psi_{j,k}(\kappa^{-1}(x))\right) \Psi_{j',k'}(\kappa^{-1}(x))dx\right| \\ &\lesssim \sum_{\tau_\nu^{j'}\subset\Omega_{j',k'}} \left|\int_{\mathbf{R}\setminus\Sigma_\nu^{j'}} \sum_{|\alpha|\le d} c_\alpha 2^{j'/2} 2^{j'}(x-x_\nu)^\alpha \tilde{\mathcal{A}}\Psi_{j,k}(\kappa^{-1}(x))dx\right|. \quad (80) \end{aligned}$$

We now give an estimate of $|(\tilde{\mathcal{A}}\Psi_{j,k})(\kappa^{-1}(x))|$.

For all $x \in \Gamma$ and $x_{jk} \in \Omega_{j,k}$, suppose that $\mathrm{dist}(x, \Omega_{j,k}) \gtrsim 2^{-j}$ and we have

$$\begin{aligned} |(\tilde{\mathcal{A}}\Psi_{j,k})(x)| &= \left|\int_{\mathbf{R}} K_{\tilde{A}}(\kappa(x), y)\Psi_{j,k}(\kappa^{-1}(y))dy\right| \\ &\lesssim \sum_{|\alpha|=\tilde{d}+1} \sup_{\kappa^{-1}(y)\in\Omega_{j,k}} |\partial_y^\alpha K_{\tilde{A}}(\kappa(x),y)| \int_{\mathbf{R}} |(y-y_0)^\alpha||\Psi_{j,k}(\kappa^{-1}(y))|dy \\ &\lesssim \frac{2^{-j(\tilde{d}+3/2)}}{[\mathrm{dist}(x,\Omega_{j,k})]^{\tilde{d}-1}} \\ &\lesssim 2^{-5j/2}[1+2^j\mathrm{dist}(x,x_{j,k})]^{1-\tilde{d}}. \end{aligned}$$

Inserting the previous estimate in the integral (80), we get

$$
\begin{aligned}
|\langle \mathcal{A}\Psi_{j,k}, \Psi_{j',k'}\rangle| &\lesssim \int_{|x-x_\nu|>c\,\mathrm{dist}(\Omega_{j,k},\Omega^s_{j',k'})} |c_\alpha||2^{j'}(x-x_\nu)|^\alpha 2^{j'/2}2^{-5j/2}[2^j|x-x_\nu|]^{1-\tilde{d}}dx \\
&\lesssim 2^{-j(\tilde{d}+3/2)}2^{j'(|\alpha|+1/2)}[\mathrm{dist}(\Omega_{j,k},\Omega^s_{j',k'})]^{|\alpha|-\tilde{d}+2}.
\end{aligned}
$$

We get the conclusion to the Lemma 5.2 due to the assumption (75). ■

With the above inequalities, we compress the stiffness matrix in two steps. First of all, for $d < \tilde{d}-3$, we define

$$
\left(A^1{}_{(j,k),(j',k')}\right)_J := \begin{cases} A_{(j,k),(j',k')}, & \text{if } \mathrm{dist}(\Omega_{j,k},\Omega_{j',k'}) \le \delta_{j,j'}, \\ 0, & \text{otherwise.} \end{cases} \tag{81}
$$

If $d < \tilde{d}-3$ and $d' \in (d, \tilde{d}-3)$, we suppose that the compression parameters satisfy

$$
\delta_{j,j'} \sim a_1 \max\{2^{-j}, 2^{-j'}, 2^{\frac{J(2(d'+1)+3)-(j+j')(\tilde{d}+d'+2)}{2(\tilde{d}+1)-3}}\}. \tag{82}
$$

We assume from now on that

$$
\tilde{d}+1 > \gamma > -\frac{3}{2}. \tag{83}
$$

We define the matrix

$$
R^1 = \left(r^1_{(j,k),(j',k')}\right)_{(j,k),(j',k')\in\nabla^J} = \left(\left(A_{(j,k),(j',k')}\right)_J - \left(A^1_{(j,k),(j',k')}\right)_J\right)_{(j,k),(j',k')}. \tag{84}
$$

We use the next intermediate lemma in order to estimate the norm of R^1.

Lemma 5.3 *Let $\tilde{d}-3 > d$, d' satisfying $d' \in (d, \tilde{d}-3)$ and $a_1 > 1$. Assume that the compression parameters are chosen such that*

$$
\delta_{j,j'} \ge a_1 \max\left\{2^{-j}, 2^{-j'}, 2^{\frac{J(2d'+5)-(j+j')(\tilde{d}+d'+2)}{2\tilde{d}-1}}\right\}. \tag{85}
$$

We therefore obtain

$$
\begin{aligned}
\sum_1 &:= \sum_{k\in\nabla_j} 2^{-j/2}2^{-(j+j')(d+1)}|r^1_{(j,k),(j',k')}| \\
&\lesssim 2^{-j'/2}a_1^{1-2\tilde{d}}2^{-J(2d+5)}2^{(j-J)(d'-d)}2^{(j'-J)(d'-d)},
\end{aligned}
$$

for $0 \le j, j' \le J$.

Proof: By the definition of $r^1_{(j,k),(j',k')}$ and using the Lemma 5.2, we have

$$\begin{aligned}\sum_1 &= \sum_{\substack{k\in\nabla_j:\\ \mathrm{dist}(\Omega_{j,k},\Omega_{j',k'})>\delta_{j,j'}}} 2^{-j/2}2^{-(j+j')(d+1)}|A_{(j,k),(j',k')}| \\ &\lesssim 2^{-j/2}2^{-(j+j')(d+1)}2^{-(j+j')(\tilde{d}+\frac{3}{2})} \\ &\times \sum_{\substack{k\in\nabla_j:\\ \mathrm{dist}(\Omega_{j,k},\Omega_{j',k'})>\delta_{j,j'}}} \mathrm{dist}(\Omega_{j,k},\Omega_{j',k'})^{-2\tilde{d}}.\end{aligned}$$

Because $\delta_{j,j'} \geq \max\{2^{-j}, 2^{-j'}\}$, the last sum above is estimated via an integral :

$$\begin{aligned}\sum_1 &\lesssim 2^{-j(d+\tilde{d}+3)}2^{-j'(d+\tilde{d}+5/2)}2^j \int_{|x|>\delta_{j,j'}} |x|^{-2\tilde{d}}dx \\ &\lesssim 2^{-j'/2}2^{-j(d+\tilde{d}+2)}2^{-j'(d+\tilde{d}+2)}(\delta_{j,j'})^{1-2\tilde{d}}.\end{aligned}$$

Using now the assumption (85) on the parameters $\delta_{j,j'}$, we find

$$\begin{aligned}\sum_1 &\lesssim 2^{-j'/2}a_1^{1-2\tilde{d}}2^{-(j+j')[d+\tilde{d}+2-(\tilde{d}+d'+2)]}2^{-J(2d'+5)} \\ &\lesssim 2^{-j'/2}a_1^{1-2\tilde{d}}2^{(j-J)(d'-d)}2^{(j'-J)(d'-d)}2^{-J[2(d+1)+3]}.\end{aligned}$$

■

We recall the Schur Lemma which allows to estimate the norm of infinite matrices (see [21] for instance).

Lemma 5.4 *Let $T = (T_{j,j'})_{j,j'\in I}$ be a matrix, $u \in l_2(I)$, $\vec{u} = (u_j)_{j\in I}$ and $s \in \mathbf{R}$. It holds*

$$\begin{aligned}\|Tu\| &\lesssim \left[\sup_{j\in I}\sum_{j'\in I}|T_{j,j'}|2^{s(j-j')}\right]^{1/2} \\ &\times \left[\sup_{j'\in I}\sum_{j\in I}|T_{j,j'}|2^{s(j'-j)}\right]^{1/2} \cdot \|u\|_{l_2(I)}.\end{aligned}$$

We use the Schur Lemma and the Lemma 5.3 to obtain an estimate of $\|R\|$.

Theorem 5.5 *Let $d' \in (d, \tilde{d}-3)$, $0 \leq t, \tilde{t} < d+1$. We pose*

$$\left(R^{1;t,\tilde{t}}_{j,j'}\right)_{j,j'} = \left(2^{-jt-j'\tilde{t}}|r^1_{(j,k),(j',k')}|\right)_{j,j'}, \tag{86}$$

and we obtain

$$\|R^{1;t,\tilde{t}}_{j,j'}\| \lesssim a_1^{1-2\tilde{d}}2^{-J(t+\tilde{t}+3)}2^{(j-J)(d'+1-t)}2^{(j'-J)(d'+1-\tilde{t})}. \tag{87}$$

Proof: We apply the Schur Lemma with $s = \frac{1}{2}$:

$$\begin{aligned}
\|R^{1;t,\tilde{t}}_{j,j'}\| &\lesssim \sup_{k'} \sum_{k} 2^{(j'-j)/2} 2^{-jt-j'\tilde{t}} |r^1| \\
&+ \sup_{k} \sum_{k'} 2^{(j-j')/2} 2^{-jt-j'\tilde{t}} |r^1| \\
&= \sum + \widetilde{\sum}.
\end{aligned}$$

We treat the first sum defined above ; the other one is estimated in the same way by interchanging the roles of j and j'. Using Lemma 5.3, we obtain

$$\sum \lesssim 2^{-j/2+j'/2} 2^{-jt-j'\tilde{t}} 2^{-(j+j')(\tilde{d}+\frac{3}{2})} \delta^{1-2\tilde{d}}_{j,j'} 2^{j}.$$

Because of the assumption (85) on the compression parameters we get :

$$\sum \lesssim a_1^{1-2\tilde{d}} 2^{-j(t-d'-1)} 2^{-j'(\tilde{t}-d'-1)} 2^{-J(2d'+5)}.$$

■

In a second step, thanks to the third estimate in Lemma 5.2, we define a second compression by the following way.

Let $d < d' < \tilde{d} - 3$ and define the matrix

$$A^c_J := \left(A^c_{(j,k),(j',k')}\right)_{(j,k),(j',k') \in \nabla^J}, \tag{88}$$

by

$$\left(A^c_{(j,k),(j',k')}\right)_J := \begin{cases} A^1_{(j,k),(j',k')} & \text{if } j' \leq j \text{ and if } \operatorname{dist}(\Omega_{j,k}, \Omega^s_{j',k'}) \leq \delta^s_{j,j'}, \\ A^1_{(j,k),(j',k')} & \text{if } j' \geq j \text{ and if } \operatorname{dist}(\Omega^s_{j,k}, \Omega_{j',k'}) \leq \delta^s_{j,j'}, \\ 0 & \text{else.} \end{cases} \tag{89}$$

If $d < d' < \tilde{d} - 3$ and $a_c > 1$, the compression parameters are such that

$$\delta^s_{j,j'} \sim a_c \max\{2^{-j}, 2^{-j'}, 2^{\frac{J(2d'+5) - \max\{j,j'\}(\tilde{d}+1) - (j+j')(d'+1)}{\tilde{d}-2}}\}. \tag{90}$$

As before, we are able to estimate the difference between the first and second compressed schemes.

Theorem 5.6 *Let $\tilde{d} - 3 > d' > d$ and pose*

$$r^c_{(j,k),(j',k')} = \left(A^1_{(j,k),(j',k')}\right)_J - \left(A^c_{(j,k),(j',k')}\right)_J, \tag{91}$$

with $dist(\Omega_{j,k}, \Omega_{j',k'}) \lesssim \min\{2^{-j}, 2^{-j'}\}$ and the compression prameters $\delta^s_{j,j'}$ satisfying (90). If we define

$$\left(R^c_{j,j'}\right) = \left(2^{-(j+j')(d+1)} |r^c_{(j,k),(j',k')}|\right), \tag{92}$$

we obtain the estimate

$$\|R^c_{(j,j')}\| \lesssim a_c 2^{-J(2d+5)} 2^{(j-J)(d'-d)} 2^{(j'-J)(d'-d)}. \tag{93}$$

Proof: We proceed as in the proof of Theorem 5.5 when estimating $\|R^{t,t'}_{j,j'}\|$. We use the Schur Lemma 5.4 and the assumption (90) to conclude. ■

6 Error estimate

In this section, we show that the compressed Galerkin scheme has the same order of convergence as the initial one. As usual, we associate to the matrices A^1_J, A^c_J the operators $\mathcal{A}^1_J, \mathcal{A}^c_J$.

We begin to estimate the difference $A^1_J - A^c_J$.

Theorem 6.1 *For $d < d' < \tilde{d} - 3$ and A^1_J defined by (81), we have*

$$\|(\mathcal{A}_J - \mathcal{A}^1_J)u\|_{-d-1} \lesssim a_1^{1-2\tilde{d}} 2^{-J(2d+5)} \|u\|_{d+1} \tag{94}$$

Proof: Using the Theorem of characterization 4.3 and the definition (86) of $R^{1;t,\tilde{t}}_{j,j'}$, we arrive at

$$\|(\mathcal{A}_J - \mathcal{A}^1_J)u\|_{-d-1} \lesssim \sum_{j,j'=-1}^{J-1} \|R^{1;d+1,d+1}_{j,j'}\| \cdot \|u\|_{d+1}. \tag{95}$$

Now we make use of the estimate (87) of Theorem 5.5 and we get

$$\|(\mathcal{A}_J - \mathcal{A}^1_J)u\|_{-d-1} \lesssim \sum_{j,j'} a_1^{1-2\tilde{d}} 2^{-J(2d+5)} 2^{(j-J)(d'-d)} 2^{(j'-J)(d'-d)} \|u\|_{d+1}. \tag{96}$$

The conclusion follows for $j, j' \leq J$. ■

By the same way, we can have such an estimate for the second compression.

Theorem 6.2 *For $d < \tilde{d} - 3$ and A^c_J defined by (89), we get the following estimate*

$$\|(\mathcal{A}_J - \mathcal{A}^c_J)u\|_{-d-1} \lesssim a_c 2^{-J(2d+5)} \|u\|_{d+1} \tag{97}$$

Proof: By Theorem 4.3 and definition (92), we write

$$\|(\mathcal{A}_J - \mathcal{A}^c_J)u\|_{-d-1} \lesssim \sum_{j,j'=-1}^{J-1} \|R^c_{j,j'}\| \cdot \|u\|_{d+1}. \tag{98}$$

Now we make use of the estimate (93) of Theorem 5.6 to obtain

$$\|(\mathcal{A}_J - \mathcal{A}^c_J)u\|_{-d-1} \lesssim a_c 2^{-J(2d+5)} \sum_{j,j'} 2^{(j-J)(d'-d)} 2^{(j'-J)(d'-d)} \|u\|_{d+1}. \tag{99}$$

■

The previous estimates allow to obtain the next Theorem of error.

Theorem 6.3 *For $i \in \{1,2\}$ let $-d_i - 4 \le s_i < \min\{\gamma_i, d_i+1\}$ with $(s_1, s_2) = (-\frac{3}{2}, -\frac{1}{2})$. Furthermore, assume that $s + s_i \le d_i + 1$. For two parameters a_1, a_c large enough, we define the compression parameters $\delta_{j,j'}, \delta^s_{j,j'}$ as in (85) and (90).*

If we assume that the exact solution satisfies $q = (q_1, q_2) \in H^{s-3/2}(\Gamma) \times H^{s-1/2}(\Gamma)$ and if we note q^c_J the solution of the (two times) compressed scheme A^c_J, we have the following optimal error estimate :

$$\|q - q^c_J\|_{H^{-2}(\Gamma)} \lesssim 2^{-sJ}\Big(\|g_0\|^2_{H^{s+3/2}(\Gamma)} + \|g_1\|^2_{H^{s+1/2}(\Gamma)}\Big)^{1/2}. \tag{100}$$

Proof: First of all, the assumption on the parameters a_1, a_c guarantees the stability of the compression procedure, in view of Theorems 6.1 and 6.2 and due to the coercivity of a.

Therefore, we can apply the first Strang Lemma in order to estimate the error. We recall that for all $s \in \mathbf{R}$,

$$\|q - q^c_J\|_s \lesssim \inf_{q_J \in S_J}\left\{\|q - q_J\|_s + \sup_{\tilde{q}_J \in S_J} \frac{|\langle(\mathcal{A}_J - \mathcal{A}^c_J)q_J, \tilde{q}_J\rangle|}{\|\tilde{q}_J\|_s}\right\}. \tag{101}$$

For the first term on the right hand, recalling that Q_J is the projection on S_J, we can use the Céa's Lemma to write

$$\begin{aligned}\|q - q_J\|_{H^{-2}(\Gamma)} &\lesssim \|q - Q_J q\|_{H^{-2}(\Gamma)}\\ &\lesssim 2^{-sJ}\|q\|_{H^{s-\frac{3}{2}}(\Gamma)\times H^{s-\frac{1}{2}}(\Gamma)},\end{aligned}$$

this last estimate being a direct consequence of Theorem 4.3.

For the second term on the right hand side of (101), we have

$$\sup_{\tilde{q}_J \in S_J} \frac{|\langle(\mathcal{A}_J - \mathcal{A}^2_J)q_J, \tilde{q}_J\rangle|}{\|\tilde{q}_J\|_s} \lesssim \|(\mathcal{A}_J - \mathcal{A}^2_J)q_J\|_{s-r},$$

which can be estimated thanks to Theorem 6.2 to get

$$\begin{aligned}\sup_{\tilde{q}_J \in S_J} \frac{|\langle(A_J - A^2_J)q_J, \tilde{q}_J\rangle|}{\|\tilde{q}_J\|_s} &\lesssim a_c\Big(2^{-2J[s-\frac{3}{2}-(-\frac{3}{2}+3)+3]}\|q_1\|^2_{s-\frac{3}{2}} + 2^{-2J[s-\frac{1}{2}-(-\frac{1}{2}+3)+3]}\|q_2\|^2_{s-\frac{1}{2}}\Big)^{1/2}\\ &\lesssim a_c 2^{-sJ}\|q\|_{H^{s-\frac{3}{2}}(\Gamma)\times H^{s-\frac{1}{2}}(\Gamma)}.\end{aligned}$$

■

Using the representation formula (16) and the continuity of operators A and B, we obtain the following error estimate for the solution of the clamped plate problem (10).

Theorem 6.4 *We note u the exact solution of the clamped plate problem and u^c_J its approximation. Under the same assumptions of Theorem 6.3, we get :*

$$\|u - u^c_J\|_{H^2_\gamma(\Gamma)} \lesssim 2^{-sJ}\left(\|g_0\|^2_{H^{s+\frac{3}{2}}(\Gamma)} + \|g_1\|^2_{H^{s+\frac{1}{2}}(\Gamma)}\right)^{1/2}. \tag{102}$$

Proof: We use the integral representation (16) of the solution u and its approximation u_J^c :

$$\begin{aligned} u(x) &= (Aq_1(x) + Bq_2(x)), \\ u_J^c(x) &= (Aq_{1J}^c(x) + Bq_{2J}^c(x)). \end{aligned}$$

And therefore we obtain

$$\|u - u_J^c\|^2_{H^2_\gamma(\Gamma)} \lesssim \|A(q_1 - q_{1J}^c)\|^2_{H^2_\gamma(\Gamma)} + \|B(q_2 - q_{2J}^c)\|^2_{H^2_\gamma(\Gamma)}.$$

Now operators A and B are of order (-3) and (-2) respectively and are continuous. Consequently we can write :

$$\begin{aligned} \|u - u_J^c\|^2_{H^2_\gamma(\Gamma)} &\lesssim \|q_1 - q_{1J}^c\|^2_{H^{-3/2}(\Gamma)} + \|q_2 - q_{2J}^c\|^2_{H^{-1/2}(\Gamma)}, \\ &\lesssim \|q - q_J^c\|^2_{H^{-2}(\Gamma)}, \\ &\lesssim 2^{-2sJ}\left(\|g_0\|^2_{H^{s+\frac{3}{2}}(\Gamma)} + \|g_1\|^2_{H^{s+\frac{1}{2}}(\Gamma)}\right), \end{aligned}$$

by applying Theorem 6.3 in the last inequality. ■

Remark 6.5 *All the previous results can be easily extended to domains* $\Omega \subset \mathbf{R}^3$. *The fundamental solution becomes*

$$E(x, y) = -\frac{1}{8\pi}|x - y|,$$

and the results of section 2 can be adapted. Using a patch-representation of the surface $\partial\Omega$, *one can construct a system of biorthogonal wavelets on the two-dimensional boundary* Γ *(see [15]). In* $\mathbf{R}^3$, *our system is allways uniquely solvable, for any curve* Γ.

7 Integral formulation of Hsiao-MacCamy

In order to give a strongly elliptic variational formulation of the problem, for any value of cap Γ, several authors studied a modified system of equations, by adding three new constraints. This work was initialized by Fichera, Hsiao, MacCamy [20], and Costabel, Stephan, Wendland [10] for an extension to a polygonal boundary. The method reads as follow.

The solution u of problem (10) admits the following representation formula

$$u(x) = -2\int_\Gamma (\partial_{y_1} G q_1 + \partial_{y_2} G q_2)\, ds_y + a_1 x_1 + a_2 x_2 + \gamma, \tag{103}$$

where $y = (y_1, y_2) \in \Gamma$, G is the fundamental solution of the bilaplacian given in (11).

For a given data $(g_0, g_1) \in H^{s+\frac{3}{2}} \times H^{s+\frac{1}{2}}$, the unknowns are the functions $(q_1, q_2) \in H^{s-\frac{1}{2}} \times H^{s-\frac{1}{2}}$ and the real numbers $a_1, a_2, a_3 \in \mathbf{R}$ which solve the system of equations

$$\begin{cases} (S + L_{11})q_1 & + & L_{12}q_2 & = & \varphi_1 - a_1 - a_3 k_1, \\ L_{21}q_1 & + & (S + L_{22})q_2 & = & \varphi_2 - a_2 - a_3 k_2, \end{cases} \tag{104}$$

where

$$Su(x) := \int_\Gamma g(x,y) u(y) ds_y, \tag{105}$$

is the single layer potential for the Laplacian with

$$g(x,y) = -\frac{1}{2\pi} \log |x - y|, \tag{106}$$

the fondamental solution (in $\mathbf{R}^2$) of Δ.

The operators L_{ij} appearing in the above system are integral operators with smooth kernels l_{ij} defined by

$$\begin{pmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{pmatrix} = -\frac{1}{2\pi} \begin{pmatrix} \frac{(x_1-y_1)^2}{|x-y|^2} + \frac{1}{2} & \frac{(x_1-y_1)(x_2-y_2)}{|x-y|^2} \\ \frac{(x_1-y_1)(x_2-y_2)}{|x-y|^2} & \frac{(x_2-y_2)^2}{|x-y|^2} + \frac{1}{2} \end{pmatrix}. \tag{107}$$

The vectors φ and k are defined by

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} \dot{g_0} \dot{x_1} + g_1 \dot{x_2} \\ \dot{g_0} \dot{x_2} - g_1 \dot{x_1} \end{pmatrix}, \quad \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix}, \tag{108}$$

where $\dot{g}$ denotes the differentiation of g with respect to the arc length. Moreover, we impose the three constraints :

$$\int_\Gamma q_1 ds_y = 0, \tag{109}$$

$$\int_\Gamma q_2 ds_y = 0, \tag{110}$$

$$\int_\Gamma q_1 \dot{x_1} ds_y + \int_\Gamma q_2 \dot{x_2} ds_y = 0. \tag{111}$$

We therefore write shortly the equations (104), (109), (110), (111) as

$$\begin{pmatrix} \mathcal{A} & \mathcal{T} \\ \Lambda & 0 \end{pmatrix} \begin{pmatrix} q \\ a \end{pmatrix} = \begin{pmatrix} \varphi \\ 0 \end{pmatrix}, \tag{112}$$

with the following definitions of the operators :

$$\mathcal{A} = \begin{pmatrix} S + L_{11} & L_{12} \\ L_{21} & S + L_{22} \end{pmatrix}, \tag{113}$$

$$\mathcal{T}a = \begin{pmatrix} a_1 + a_3 k_1 \\ a_2 + a_3 k_2 \end{pmatrix}, \tag{114}$$

$$\Lambda q = \begin{pmatrix} \int_\Gamma q_1 ds \\ \int_\Gamma q_2 ds \\ \int_\Gamma (q_1 \dot{x_1} + q_2 \dot{x_2}) ds \end{pmatrix}. \tag{115}$$

Remark 7.1 *The constant γ in (103) can be easily found by imposing that $u(x_0) = g_0(x_0)$, for a particular point x_0 on the boundary Γ.*

For two real numbers s_1, s_2, let us introduce the notation :

$$\mathcal{H}^{s_1,s_2}(\Gamma) := H^{s_1}(\Gamma) \times H^{s_2}(\Gamma) \times \mathbf{R}^3. \tag{116}$$

The operator involved in (112) has the following property (see [10]).

Lemma 7.2 *The operator $\mathcal{A}$ is a strongly elliptic pseudodifferential operator of order (-1) and the operator*

$$\mathcal{B} := \begin{pmatrix} \mathcal{A} & \mathcal{T} \\ \Lambda & 0 \end{pmatrix} : \mathcal{H}^{s-\frac{1}{2},s-\frac{1}{2}}(\Gamma) \longrightarrow \mathcal{H}^{s+\frac{1}{2},s+\frac{1}{2}}(\Gamma) \tag{117}$$

is bijective, for all $s \in \mathbf{R}$.

For the sake of brevity, we define $\Phi = (q, a) = (q_1, q_2, a_1, a_2, a_3)$. The variational formulation of problem (112) reads as :

Find $\Phi \in \mathcal{H}^{-1/2,-1/2}(\Gamma)$ solution of

$$b(\Phi, \Phi') := \langle \mathcal{B}\Phi, \Phi' \rangle_{\mathcal{H}^{1/2,1/2}(\Gamma)\times\mathcal{H}^{-1/2,-1/2}(\Gamma)}, \forall \Phi' \in \mathcal{H}^{-1/2,-1/2}(\Gamma). \tag{118}$$

In view of Lemma 7.2, the bilinear form b is symmetric, continuous and coercive on $\mathcal{H}^{-1/2,-1/2}(\Gamma)$.

We therefore write in detail the Galerkin discretization of the problem (112). As in section 3, for a given mesh Δ_J, we define as in (39) the approximation space

$$S_J := S_J^{d_1} \times S_J^{d_2} \subset H^{s-\frac{1}{2}} \times H^{s-\frac{1}{2}}, \quad j = 0, \ldots, J, \tag{119}$$

and with the assumption $d_1, d_2 \geq s - 3/2$. A priori, one chooses $d_1 = d_2$.

Let us make use of the following notations : $q = (q_1, q_2)$, $a = (a_1, a_2, a_3)$ for the unknowns and $q_J = (q_{1J}, q_{2J})$, $a_J = (a_{1J}, a_{2J}, a_{3J})$ for their respective approximations ; if we note $q'_J = (q'_{1J}, q'_{2J})$ and $a'_J = (a'_{1J}, a'_{2J}, a'_{3J})$, with $\Phi'_J = (q'_J, a'_J)$ the Galerkin scheme reads as :

Find $\Phi_J = (q_J, a_J) \in S_J^{d_1} \times S_J^{d_2} \times \mathbf{R}^3$ such that

$$b(\Phi_J, \Phi'_J) := \left\langle \left(\begin{array}{cc} \mathcal{A} & \mathcal{T} \\ \Lambda & 0 \end{array} \right) \left(\begin{array}{c} q_J \\ a_J \end{array} \right), \left(\begin{array}{c} q'_J \\ a_J \end{array} \right) \right\rangle = \left\langle \left(\begin{array}{c} \varphi \\ 0 \end{array} \right), \left(\begin{array}{c} q'_J \\ a_J \end{array} \right) \right\rangle, \qquad (120)$$

for all $\Phi'_J \in S_J^{d_1} \times S_J^{d_2} \times \mathbf{R}^3$.

Using spline basis, we have the error estimate [8] :

Lemma 7.3 *Let $\Phi = (q, a) \in \mathcal{H}^{s-\frac{1}{2}, s-\frac{1}{2}}(\Gamma)$ be the exact solution of problem (112) with data $(g_0, g_1) \in H^{s+\frac{3}{2}}(\Gamma) \times H^{s+\frac{1}{2}}(\Gamma)$. For $d_1, d_2 \geq s - 3/2$, we get*

$$\|q - q_J\|_{H^{-1/2}(\Gamma)\times H^{-1/2}(\Gamma)} + |a - a_J| \lesssim 2^{-sJ} \left(\|g_0\|^2_{H^{s+\frac{3}{2}}(\Gamma)} + \|g_1\|_{H^{s+\frac{1}{2}}(\Gamma)} \right)^{1/2}. \qquad (121)$$

Proof: Let us define $\Phi = (q_1, q_2, a_1, a_2, a_3) \in \mathcal{H}^{s-\frac{1}{2}, s-\frac{1}{2}}(\Gamma)$ and $\Phi_J = (q_{1J}, q_{2J}, a_{1J}, a_{2J}, a_{3J})$ its Galerkin approximation. If $|a|$ denotes the usual euclidian norm on $\mathbf{R}^3$, a norm on $\mathcal{H}^{t_1,t_2}(\Gamma)$ is given by

$$\|\Phi\|_{\mathcal{H}^{-\frac{1}{2},-\frac{1}{2}}(\Gamma)} = \left(\|q_1\|^2_{H^{-\frac{1}{2}}(\Gamma)} + \|q_2\|^2_{H^{-\frac{1}{2}}(\Gamma)} + |a|^2 \right)^{1/2}.$$

Therefore, due to the coercivity of the bilinear form a, we have by Céa's Lemma :

$$\begin{aligned} \|\Phi - \Phi_J\|^2_{\mathcal{H}^{-\frac{1}{2},-\frac{1}{2}}} &\lesssim \|\Phi - Q_J\Phi\|^2_{\mathcal{H}^{-\frac{1}{2},-\frac{1}{2}}} \\ &\lesssim \|q_1 - Q_J q_1\|^2_{H^{-\frac{1}{2}}} + \|q_2 - Q_J q_2\|^2_{H^{-\frac{1}{2}}} + |a - \overline{Q_J} a|^2, \end{aligned}$$

where Q_J is the $L^2(\Gamma)^2$-projection on S_J and $\overline{Q_J}$ is the scalar projection. Now applying Theorem 4.3 of section 4, we have respectively

$$\begin{aligned} \|q_1 - Q_J q_1\|^2_{H^{-\frac{1}{2}}} &\lesssim 2^{-2sJ} \|q_1\|^2_{H^{s-\frac{1}{2}}}, \\ \|q_2 - Q_J q_2\|^2_{H^{-\frac{1}{2}}} &\lesssim 2^{-2sJ} \|q_2\|^2_{H^{s-\frac{1}{2}}}. \end{aligned}$$

The estimate for $|a - \overline{Q_J} a|^2$ is deduced from the equation (104), replacing a_i by $(a_i - \overline{Q_J} a_i)$, $i = 1, 2, 3$. ■

Now we describe the wavelet Galerkin method which allows to compress the matrix A associated to operator $\mathcal{A}$. We keep the same construction of the biorthogonal wavelets as in section 4 and obtain a basis which characterizes $H^{s-\frac{1}{2}}(\Gamma) \times H^{s-\frac{1}{2}}(\Gamma)$. Namely, we define

$$\{\Psi_{j,k}\}_{k\in\nabla_j} := \{\psi^1_{j,k}\}_{k\in\nabla^1_j} \cup \{\psi^2_{j,k}\}_{k\in\nabla^2_j}, \qquad (122)$$

with ψ^1, ψ^2 defined by (58)-(59). We suppose that $\gamma \in \{\gamma_1, \gamma_2\}$ satisfies

$$\gamma > -\frac{1}{2}. \qquad (123)$$

Consequently, we get the same Theorem of characterization as Theorem 4.3 in the range $s \in (-\frac{1}{2}, -\frac{1}{2}+\sigma)$, for $\sigma \in \mathbf{R}_+$.

If we define the stiffness matrix related to operator $\mathcal{A}$ as

$$A_J := (\langle \mathcal{A}\Psi_{j,k}, \Psi_{j',k'}\rangle)_{(j,k),(j',k')\in\nabla}, \tag{124}$$

we introduce the diagonal matrix

$$\left(\overline{D_{s,j}}\right)_{(j,k),(j',k')} = 2^{-sj}\delta_{(j,k),(j',k')} \tag{125}$$

and we pose

$$D_{s,j} = \begin{pmatrix} \overline{D_{s,j}} & 0 \\ 0 & \overline{D_{s,j}} \end{pmatrix}.$$

Consequently, we get

$$\text{Cond}\left(D_{1/2,J}A_J D_{1/2,J}\right) \lesssim 1. \tag{126}$$

Proof: The proof is similar to these of Proposition 5.1. ■

The advantage of using wavelets is also the possibility to compress the stiffness matrix of the Galerkin scheme. In fact, regarding more precisely the operator $\mathcal{A}$ defined by (113), we show that the matrices related to the operators S and L_{ij} can be compressed thanks to the particular form of their kernels.

First of all, the kernel $g(x,y) = -\frac{1}{2\pi}\log|x-y|$ of the single layer potential operator S satisfies the property

$$\left|\frac{\partial^{|\alpha|+|\beta|}g(x,y)}{\partial x^\alpha \partial y^\beta}\right| \lesssim \frac{1}{|x-y|^{|\alpha|+|\beta|}},$$

which fits well in our compression procedure.

Concerning operators L_{ij}, $i,j = 1,2$, we can write the corresponding kernels in the form

$$l_{ij}(x-y) = \frac{p(x_1-y_1, x_2-y_2)}{|x-y|^2} + C,$$

when p is a polynomial of degree 2 and C is a real constant. If we pose $t = (x-y)$ and $l_{ij}(t) = p(t)/|t|^2$, we can easily show the

Lemma 7.4 *For the multi-indices α, β, the next estimate holds :*

$$\left|\frac{\partial^{|\alpha|+|\beta|}l_{ij}(x-y)}{\partial x^\alpha \partial y^\beta}\right| \lesssim \frac{1}{|x-y|^{|\alpha|+|\beta|}}. \tag{127}$$

This result show that the kernels of S and operators L_{ij} for $i,j \in \{1,\ldots,4\}$ have the same decreasing property. Because $\mathcal{A}$ is of order (-1) and applying a similar proof as in Lemma 5.2, we have :

Lemma 7.5 *The following estimates hold :*

1. *For* $(\tilde{d}+1) > 0$ *and* $dist(\Omega_{j,k};\Omega_{j',k'}) > 0$ *we get*

$$\left|A_{(j,k),(j',k')}\right| \lesssim \frac{2^{-(j+j')(\frac{1}{2}+\tilde{d})}}{dist(\Omega_{j,k};\Omega_{j',k'})^{2(\tilde{d}+1)}}. \tag{128}$$

2. *Let* $j' \leq j$, $0 \leq d < \tilde{d}-1$ *and* $\gamma > -\frac{1}{2}$. *We note* $\Omega_{j',k'} = \cup_{\nu=1}^{n_\nu}\{\Sigma_\nu^{j'}\}$ *where* $s_\nu^{j'} := \Sigma_\nu^{j'-1} \cup \Sigma_\nu^{j'}$ *are the points of* $\Omega_{j',k'}$ *on which the wavelet* $\Psi_{j',k'}$ *is not smooth. Suppose now that*

$$\Psi_{j',k'}(\kappa^{-1})|\Sigma_\nu^{j'} \in \mathbf{P}_d.$$

If we assume that

$$2^{-j'} \gtrsim dist(\Omega_{j,k},\Omega^s_{j',k'}) \gtrsim 2^{-j},$$

we obtain

$$|\langle \mathcal{A}\Psi_{j,k},\Psi_{j',k'}\rangle| \lesssim 2^{-j(\tilde{d}+1/2)}2^{j'/2}dist(\Omega_{j,k},\Omega^s_{j',k'})^{-\tilde{d}}. \tag{129}$$

With this property, we define the compressed matrices A^1 and A^c as in (81)-(89) with two compression parameters satisfying, for $\tilde{d}-1 > d$, $d' \in (d,\tilde{d}-1)$ and $a_1 > 1$ respectively :

$$\begin{aligned}
\delta_{j,j'} &\geq a_1 \max\{2^{-j},2^{-j'},2^{\frac{J(2d'+3)-(j+j')(\tilde{d}+d'+2)}{2\tilde{d}+1}}\}, &(130)\\
\delta^s_{j,j'} &\geq a_c \max\{2^{-j},2^{-j'},2^{\frac{J(2d'+3)-\max\{j,j'\}(\tilde{d}+1)-(j+j')(d'+1)}{\tilde{d}}}\}. &(131)
\end{aligned}$$

Concerning the error estimates, we have an equivalent of Theorems 6.1 and 6.2 in the sense of

Theorem 7.6 *For* $d < d' < \tilde{d}-1$, A^1_J *and* A^c_J *defined as in (81) and (89) with compression parameters defined as above, we have :*

$$\|(A_J - A^1_J)u\|_{-d-1} \lesssim a_1^{-1-2\tilde{d}}2^{-J(2d+3)}\|u\|_{d+1}, \tag{132}$$

and

$$\|(A^1_J - A^c_J)u\|_{-d-1} \lesssim a_c 2^{-J(2d+3)}\|u\|_{d+1}. \tag{133}$$

The proof is similar to these of Theorems 6.1 and 6.2.

We conclude by giving the next error estimate.

Theorem 7.7 *For* $i \in \{1,2\}$, *let* $-d_i-2 \leq -\frac{1}{2} < \min\{\gamma_i, d_i+1\}$, $s > 0$ *and* $s-\frac{1}{2} \leq d_i+1$. *For* a_1, a_c *sufficiently large, we obtain :*

$$\|q - q^c_J\|_{H^{-\frac{1}{2}}(\Gamma)\times H^{-\frac{1}{2}}(\Gamma)} \lesssim 2^{-sJ}\left(\|g_0\|^2_{H^{s+\frac{3}{2}}(\Gamma)} + \|g_1\|^2_{H^{s+\frac{1}{2}}(\Gamma)}\right)^{1/2}. \tag{134}$$

Proof: We proceed as in the proof of Theorem 6.3 and write the first Strang Lemma (101).

For the first term on the right hand side, we can use the Céa's Lemma to write

$$\begin{aligned} \|q-q_J\|_{-\frac{1}{2},-\frac{1}{2}} &\lesssim \|q-Q_Jq\|_{-\frac{1}{2},-\frac{1}{2}} \\ &\lesssim 2^{-sJ}\|q\|_{s-\frac{1}{2},s-\frac{1}{2}}, \end{aligned}$$

by using Theorem 4.3.

For the second term on the right hand side of the Strang Lemma, we have

$$\begin{aligned} \sup_{\tilde{q}_J\in S_J}\frac{|\langle (A_J-A_J^c)q_J,\tilde{q}_J\rangle|}{\|\tilde{q}_J\|_s} &\lesssim \|(A_J-A_J^c)q_J\|_{\frac{1}{2}}, \\ &\lesssim a_c 2^{-J[s-\frac{1}{2}-(-\frac{1}{2}+1)+1]}\|q\|_{s-\frac{1}{2},s-\frac{1}{2}} \\ &\lesssim a_c 2^{-sJ}\|q\|_{s-\frac{1}{2},s-\frac{1}{2}}. \end{aligned}$$

■

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