Numerische lineare Algebra WS 2018/2019

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Homepage zur Übung: `https://www.tu-chemnitz.de/mathematik/wire/WS1819/nla.php`

11. Some estimations

1. Let $B \in \mathbb{R}^{n\times n}$ mit $||B|| < 1$. Proof the existence of $(I+B)^{-1}$ and that the estimation

$$\frac{1}{1+||B||} \leq ||(I+B)^{-1}|| \leq \frac{1}{1-||B||}$$

holds.

2. Let $A, \tilde{A} \in \mathbb{R}^{n\times n}$ and A regular. Moreover holds $||A^{-1}|| \leq \beta$, $||A-\tilde{A}|| \leq \alpha$ and $\alpha\beta < 1$.
Proof that in this case, $\tilde{A}$ is regular too and

$$||\tilde{A}^{-1}|| \leq \frac{1}{1-\alpha\beta}||A^{-1}|| \qquad \text{as well} \qquad ||A^{-1}-\tilde{A}^{-1}|| \leq \frac{\beta^2}{1-\alpha\beta}||A-\tilde{A}||$$

holds.

3. Let $A \in \mathbb{R}^{n\times n}$ be regular. What can be said about the environment of A ?

4. We consider a diagonalisable matrix $A \in \mathbb{C}^{n\times n}$ and a disturbance $\delta A \in \mathbb{C}^{n\times n}$.
Proof that every eigenvalue η in the spectrum of $A+\delta A$ has a maximum distance of

$$\min_{\lambda_i EW von A} |\eta - \lambda_i| \leq ||S^{-1}\delta A S|| \leq \kappa(S)||\delta A||$$

with condition number $\kappa(S)$ and $S = [v_1, \cdots v_n]$ eigenvectors of A.
What happens in case $A = A^*$?.

5. Let T be a tridiagonal matrix

$$T = \begin{bmatrix} \alpha_1 & \beta_1 & & & \\ \gamma_1 & \ddots & \ddots & & \\ & \ddots & & & \beta_{n-1} \\ & & & \gamma_{n-1} & \alpha_n \end{bmatrix}$$

with $\alpha_i \in \mathbb{R}$ and $\gamma_i = \bar{\beta}_i$, such that T is hermitian. Furthermore $\gamma_i \neq 0, i = 1 \cdots n-1$.
Proof that all eigenvalues of T are simple and real.
What happens if the condition $\gamma_i \neq 0, i = 1 \cdots n-1$ is hurt ?