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Homepage zur Übung: `https://www.tu-chemnitz.de/mathematik/wire/WS1819/nla.php`

8. Projections

1. Specify the idempotent matrices $P_{\mathcal{R},\mathcal{S}}$ for the *Projektion onto $\mathcal{R}$ orthogonal to $\mathcal{S}$*:
   1.1. $\mathcal{R} = span\{(1,0)^\top\}$ und $\mathcal{S} = span\{(1,1)^\top\}$,
   1.2. $\mathcal{R} = span\{(1,2)^\top\} = \mathcal{S}$.
2. Let $\mathbf{u} \in \mathbb{R}^n$ with $\|\mathbf{u}\| = 1$ and $P = \mathbf{u}\mathbf{u}^\top$ as soon as $Q = I_n - \mathbf{u}\mathbf{u}^\top$.
   2.1. Determine image and kernel of P and Q and proof that both are orthogonal projectors.
   2.2. Show that $rank(P) = 1$ und $rank(Q) = n - 1$. Determine all eigenvalues and appropriate subspaces of P and Q.
3. Let $\mathcal{R}$ and $\mathcal{S}^\perp$ non-empty, complementary subspaces of $\mathbb{C}^n$. Prrof the following statements about projections.
   3.1. $I_n - P_{\mathcal{R},\mathcal{S}} = P_{\mathcal{S}^\perp,\mathcal{R}^\perp}$.
   3.2. $P_{\mathcal{R},\mathcal{S}}^\top$ ist a projection.
   3.3. Every projection $P = P_{\mathcal{R},\mathcal{S}}$ is *diagonalisable*, i.e., it exists a regular matrix $T \in \mathbb{C}^{n\times n}$, such that TPT^{-1} is a diagonal matrix.
   To which diagonal matrix is P similar?
   3.4. For every projection $P_{\mathcal{R},\mathcal{S}}$ there is $\|P_{\mathcal{R},\mathcal{S}}\| \geq 1$. Moreover equality is given if and only if $P_{\mathcal{R},\mathcal{S}}$ is an orthogonal projection.
   3.5. A projection $P_{\mathcal{R},\mathcal{S}}$ is symmetric (self adjoint respective to an inner product) if and only if it is an orthogonal projection.