

# Markowitz Asset Allocation

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*Tue Jun 26 13:27:03 2018*

```
library(quantmod)

## Loading required package: xts
## Loading required package: zoo
##
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##   as.Date, as.Date.numeric
## Loading required package: TTR
## Version 0.4-0 included new data defaults. See ?getSymbols.
## Learn from a quantmod author: https://www.datacamp.com/courses/importing-and-managing-financial-data
rm(list = ls())

#directory for graphics
if (!dir.exists("./pics-out")) dir.create(path="pics-out")

titles = c("DAX","RWE","gold","oil","$/€")

all = c(9798.11,12.870, 981.49, 34.9, 0.9228,
        9495.40, 10.540, 1030.49, 33.08, 0.9197,
        9965.51, 11.375, 1144.73, 33.77, 0.8787,
        10038.97, 13.045, 1137.17, 37.04, 0.8729,
        10262.74, 11.765, 1192.35, 43.71, 0.8983,
        9680.09, 14.190, 1121.85, 45.81, 0.9005,
        10337.50, 15.905, 1222.25, 46.04, 0.8949,
        10592.69, 14.665, 1244.67, 39.78, 0.8962,
        10511.02, 15.335, 1204.53, 43.35, 0.8896,
        10665.01, 14.460, 1212.10, 46.15, 0.9107,
        10640.30, 11.860, 1180.53, 44.95, 0.9444,
        11481.06, 11.815, 1082.17, 47.72, 0.9509,
        11599.01, 11.800, 1081.43, 52.69, 0.9494)

S = matrix(all,ncol=5,byrow=TRUE)
nr.assets = dim(S)[2]
n = dim(S)[1]
colnames(S) = titles

time = seq(from=as.Date('2016-01-01'),by="month",length.out=n)

(S.xts = as.xts(S,frequency = 12, order.by = time))
```

```
##           DAX      RWE      gold  oil    $/\200
## 2016-01-01 9798.11 12.870  981.49 34.90 0.9228
## 2016-02-01 9495.40 10.540 1030.49 33.08 0.9197
## 2016-03-01 9965.51 11.375 1144.73 33.77 0.8787
## 2016-04-01 10038.97 13.045 1137.17 37.04 0.8729
## 2016-05-01 10262.74 11.765 1192.35 43.71 0.8983
## 2016-06-01 9680.09 14.190 1121.85 45.81 0.9005
## 2016-07-01 10337.50 15.905 1222.25 46.04 0.8949
## 2016-08-01 10592.69 14.665 1244.67 39.78 0.8962
## 2016-09-01 10511.02 15.335 1204.53 43.35 0.8896
## 2016-10-01 10665.01 14.460 1212.10 46.15 0.9107
## 2016-11-01 10640.30 11.860 1180.53 44.95 0.9444
## 2016-12-01 11481.06 11.815 1082.17 47.72 0.9509
## 2017-01-01 11599.01 11.800 1081.43 52.69 0.9494
```

```
##### help functions #####
```

```
#normalising function for rescaling assets (S0=1)
normalise_series = function(xdat) xdat / coredata(xdat)[1]
normalise_assetmatrix = function(assets){
  n = dim(assets)[1] #sample size
  nr = dim(assets)[2] #assets

  S.scaled = assets

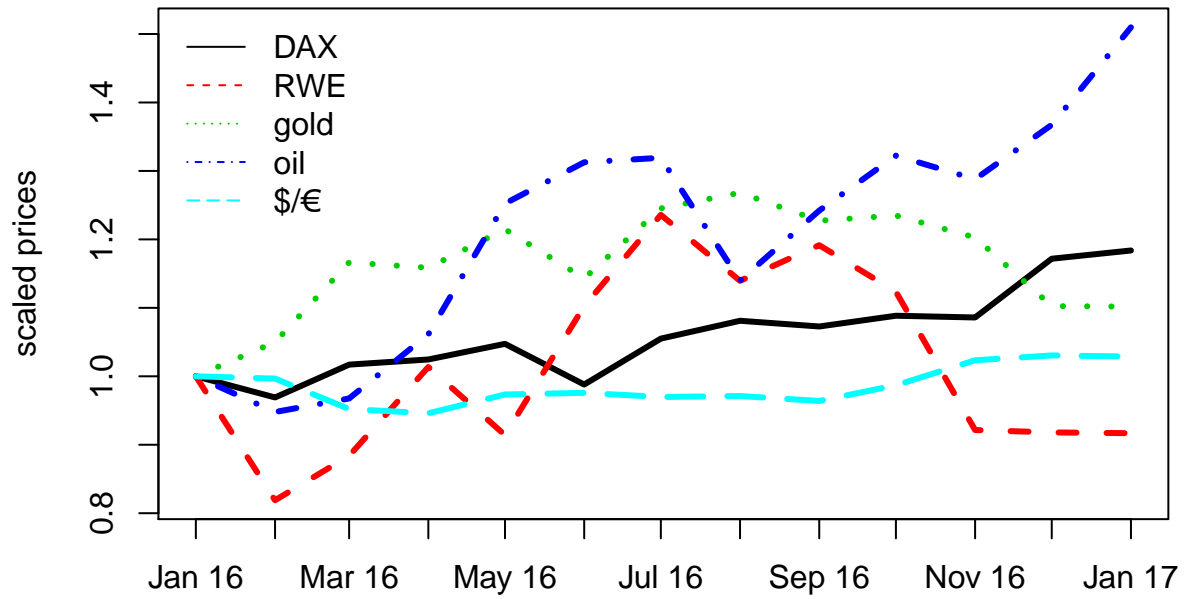
  for (k in (1:nr)){
    S.scaled[,k] = normalise_series(assets[,k])
  }
  return(S.scaled)
}
```

```
#####
(S.scaled = normalise_assetmatrix(S.xts))
```

```
##           DAX      RWE      gold      oil      $/\200
## 2016-01-01 1.0000000 1.0000000 1.000000 1.0000000 1.0000000
## 2016-02-01 0.9691053 0.8189588 1.049924 0.9478510 0.9966407
## 2016-03-01 1.0170849 0.8838384 1.166319 0.9676218 0.9522107
## 2016-04-01 1.0245823 1.0135975 1.158616 1.0613181 0.9459254
## 2016-05-01 1.0474204 0.9141414 1.214837 1.2524355 0.9734504
## 2016-06-01 0.9879548 1.1025641 1.143007 1.3126074 0.9758344
## 2016-07-01 1.0550504 1.2358197 1.245301 1.3191977 0.9697659
## 2016-08-01 1.0810952 1.1394716 1.268143 1.1398281 0.9711747
## 2016-09-01 1.0727600 1.1915307 1.227246 1.2421203 0.9640225
## 2016-10-01 1.0884762 1.1235431 1.234959 1.3223496 0.9868877
## 2016-11-01 1.0859543 0.9215229 1.202794 1.2879656 1.0234070
## 2016-12-01 1.1717627 0.9180264 1.102579 1.3673352 1.0304508
## 2017-01-01 1.1838008 0.9168609 1.101825 1.5097421 1.0288253
```

```
### chart
```

```
matplot(time,S.scaled,type='l',xaxt="n",col=1:nr.assets,xlab="",ylab="scaled prices",lwd=3)
axis(side=1, at=time, labels = format(time,"%b %y"))
legend("topleft", titles, col=1:nr.assets, lty=1:nr.assets,bty="n",lwd=1)
```



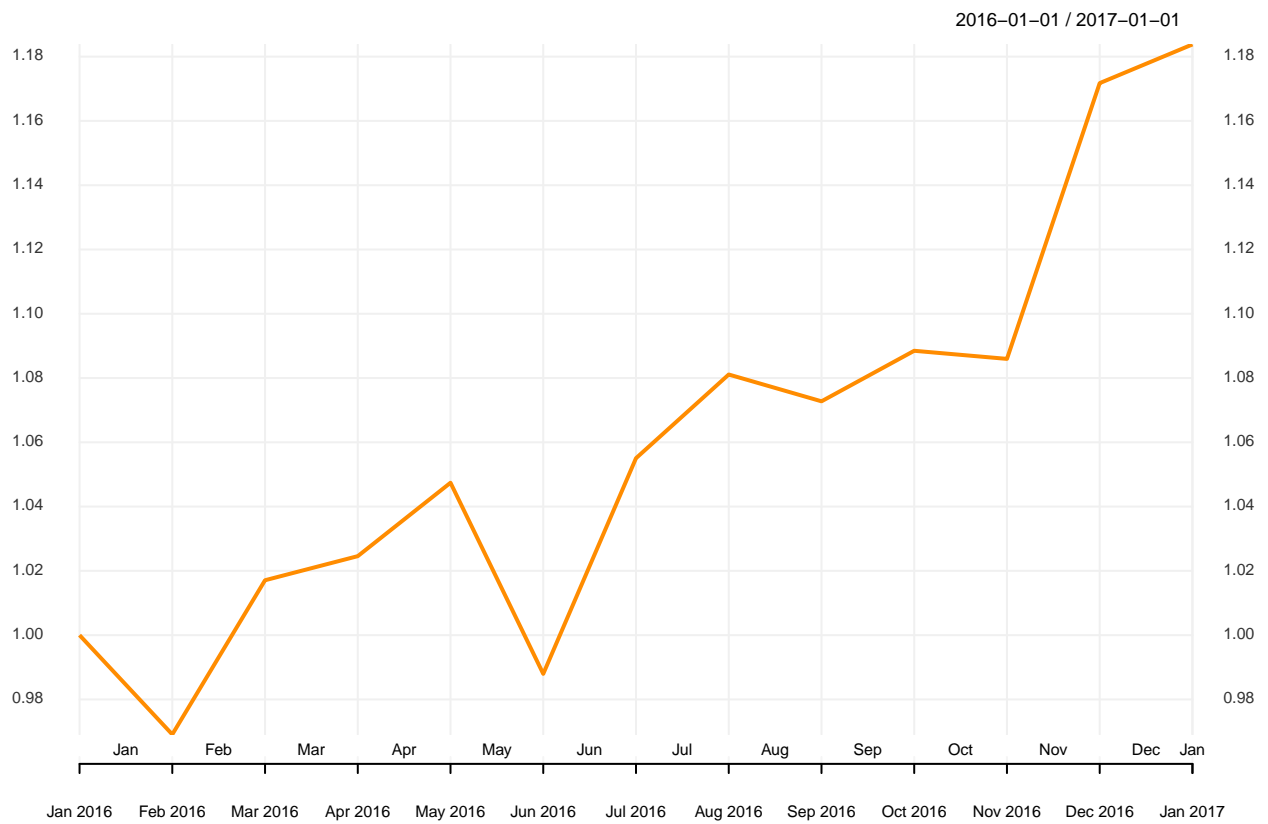
```

### using ts / quantmod charts

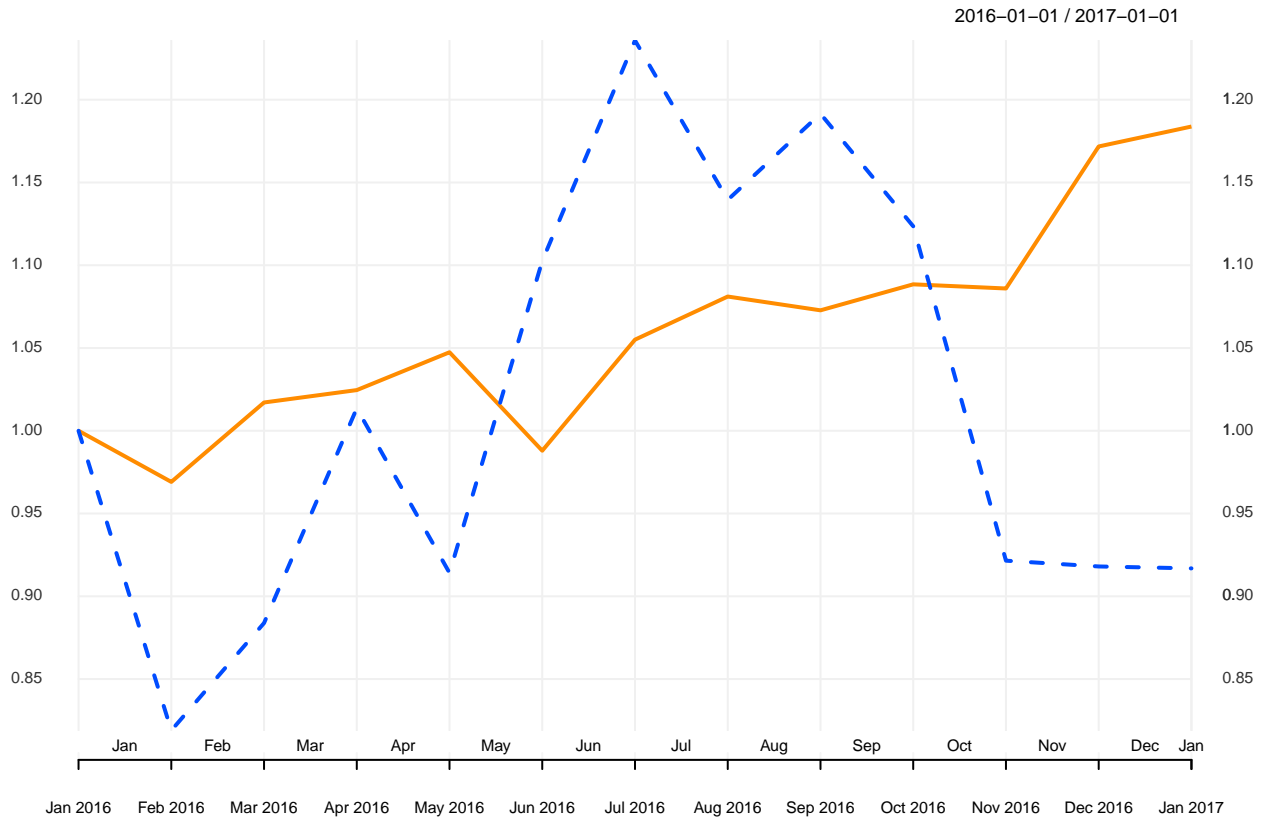
all.cols = topo.colors(nr.assets, alpha = 1)

chart_Series(S.scaled[,1], lwd=2, name="")

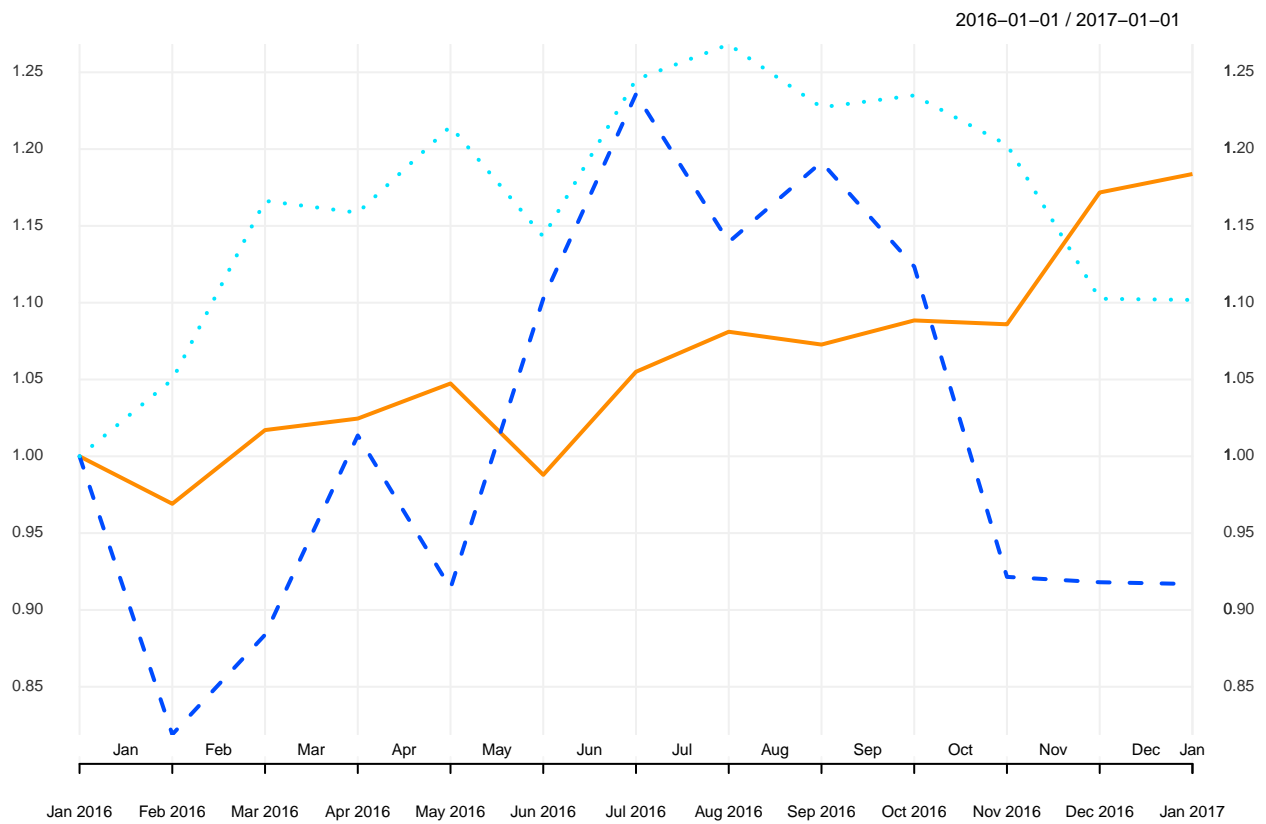
```



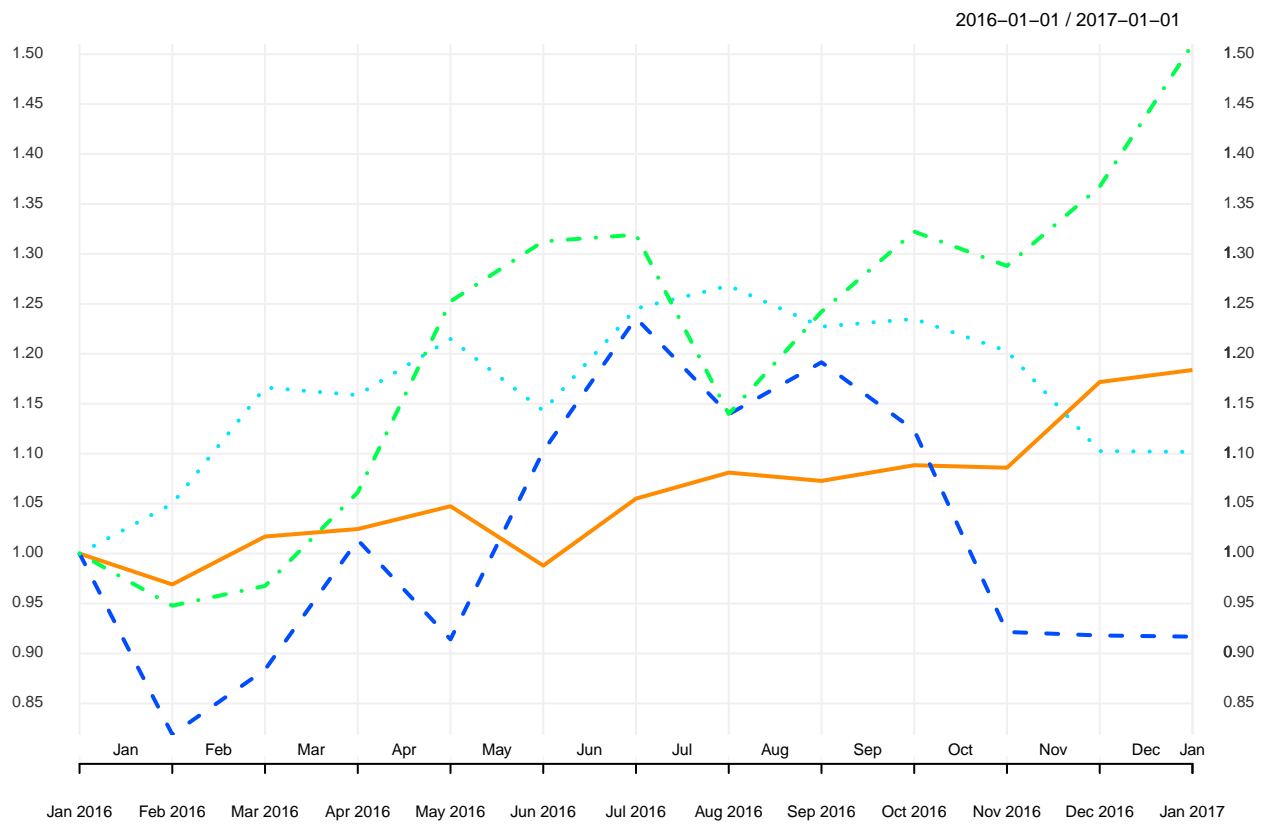
```
add_TA(S.scaled[,2], on = 1, col = all.cols[2], lty = 2, lwd=2)
```



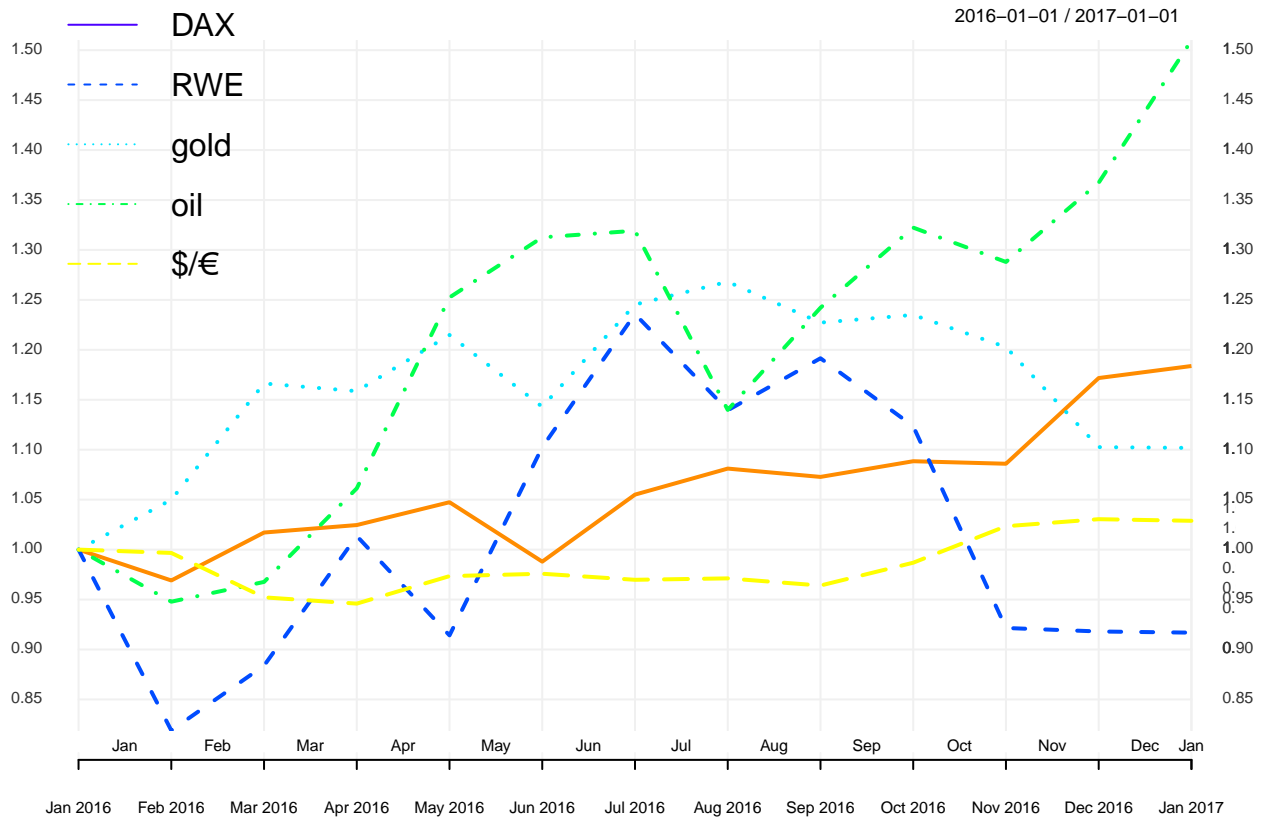
```
add_TA(S.scaled[,3], on = 1, col = all.cols[3], lty = 3,lwd=2)
```



```
add_TA(S.scaled[,4], on = 1, col = all.cols[4], lty = 4, lwd=2)
```



```
add_TA(S.scaled[,5], on = 1, col = all.cols[5], lty = 5,lwd=2)
legend("topleft", titles, col=all.cols, lty=1:nr.assets,bty="n",lwd=1)
```



#####

```
#alternative individual computation
DAX = as.xts(S[,1],frequency = 12, order.by = time)
RWE = as.xts(S[,2],frequency = 12, order.by = time)
GOLD = as.xts(S[,3],frequency = 12, order.by = time)
OIL = as.xts(S[,4],frequency = 12, order.by = time)
USEUR = as.xts(S[,5],frequency = 12, order.by = time)
```

```
DAX.scaled = normalise_series(DAX)
RWE.scaled = normalise_series(RWE)
GOLD.scaled = normalise_series(GOLD)
OIL.scaled = normalise_series(OIL)
USEUR.scaled = normalise_series(USEUR)
```

#####

```
#matrix of annulized returns
(Xi = 12*diff(log(S)))
```

##	DAX	RWE	gold	oil	\$/\200
## [1,]	-0.37658447	-2.39665774	0.584614453	-0.64269551	-0.04037996
## [2,]	0.57987193	0.91488506	1.261612609	0.24772757	-0.54724785
## [3,]	0.08813265	1.64384345	-0.079512978	1.10910690	-0.07947049



```
## [4,] 0.26454410 -1.23931076 0.568601158 1.98694199 0.34419704
## [5,] -0.70138394 2.24890163 -0.731364497 0.56310599 0.02935292
## [6,] 0.78848234 1.36915241 1.028571770 0.06009811 -0.07485821
## [7,] 0.29263296 -0.97403787 0.218124145 -1.75375573 0.01741947
## [8,] -0.09287892 0.53608914 -0.393371822 1.03130916 -0.08870015
## [9,] 0.17452871 -0.70501897 0.075179318 0.75108309 0.28129933
## [10,] -0.02783533 -2.37857788 -0.316690828 -0.31615440 0.43603525
## [11,] 0.91260051 -0.04561780 -1.043942468 0.71759804 0.08230919
## [12,] 0.12265234 -0.01524455 -0.008208542 1.18890104 -0.01894438
```

```
#script table 3.2 annualized returns
```

```
Xi * 100
```

```
##          DAX          RWE          gold          oil          $/\200
## [1,] -37.658447 -239.665774  58.4614453 -64.269551 -4.037996
## [2,]  57.987193  91.488506 126.1612609  24.772757 -54.724785
## [3,]  8.813265 164.384345  -7.9512978 110.910690 -7.947049
## [4,] 26.454410 -123.931076  56.8601158 198.694199 34.419704
## [5,] -70.138394 224.890163 -73.1364497  56.310599  2.935292
## [6,] 78.848234 136.915241 102.8571770  6.009811 -7.485821
## [7,] 29.263296 -97.403787  21.8124145 -175.375573  1.741947
## [8,] -9.287892  53.608914 -39.3371822 103.130916 -8.870015
## [9,] 17.452871 -70.501897  7.5179318  75.108309 28.129933
## [10,] -2.783533 -237.857788 -31.6690828 -31.615440 43.603525
## [11,] 91.260051 -4.561780 -104.3942468  71.759804  8.230919
## [12,] 12.265234 -1.524455  -0.8208542 118.890104 -1.894438
```

```
#factor 12 for annualized returns:
```

```
1/as.numeric(time[n]-time[1])*as.numeric(time[2]-time[1])
```

```
## [1] 0.08469945
```

```
1/12
```

```
## [1] 0.08333333
```

```
1/as.numeric(time[n]-time[1])*as.numeric(diff(time))
```

```
## [1] 0.08469945 0.07923497 0.08469945 0.08196721 0.08469945 0.08196721
```

```
## [7] 0.08469945 0.08469945 0.08196721 0.08469945 0.08196721 0.08469945
```

```
1/12
```

```
## [1] 0.08333333
```

```
#same as S.scaled after removing first line (NA)!
```

```
12*diff(log(S.scaled))[-1,]
```

```
##          DAX          RWE          gold          oil          $/\200
## 2016-02-01 -0.37658447 -2.39665774  0.584614453 -0.64269551 -0.04037996
## 2016-03-01  0.57987193  0.91488506  1.261612609  0.24772757 -0.54724785
## 2016-04-01  0.08813265  1.64384345 -0.079512978  1.10910690 -0.07947049
## 2016-05-01  0.26454410 -1.23931076  0.568601158  1.98694199  0.34419704
## 2016-06-01 -0.70138394  2.24890163 -0.731364497  0.56310599  0.02935292
## 2016-07-01  0.78848234  1.36915241  1.028571770  0.06009811 -0.07485821
## 2016-08-01  0.29263296 -0.97403787  0.218124145 -1.75375573  0.01741947
## 2016-09-01 -0.09287892  0.53608914 -0.393371822  1.03130916 -0.08870015
## 2016-10-01  0.17452871 -0.70501897  0.075179318  0.75108309  0.28129933
## 2016-11-01 -0.02783533 -2.37857788 -0.316690828 -0.31615440 0.43603525
```

```
## 2016-12-01 0.91260051 -0.04561780 -1.043942468 0.71759804 0.08230919
## 2017-01-01 0.12265234 -0.01524455 -0.008208542 1.18890104 -0.01894438
```

```
#or individual
```

```
monthlyReturn(DAX,type = 'log')
```

```
##          monthly.returns
## 2016-01-01 0.000000000
## 2016-02-01 -0.031382039
## 2016-03-01 0.048322661
## 2016-04-01 0.007344388
## 2016-05-01 0.022045341
## 2016-06-01 -0.058448662
## 2016-07-01 0.065706862
## 2016-08-01 0.024386080
## 2016-09-01 -0.007739910
## 2016-10-01 0.014544059
## 2016-11-01 -0.002319611
## 2016-12-01 0.076050042
## 2017-01-01 0.010221029
```

```
monthlyReturn(DAX,type = 'arithmetic')
```

```
##          monthly.returns
## 2016-01-01 0.000000000
## 2016-02-01 -0.030894734
## 2016-03-01 0.049509236
## 2016-04-01 0.007371424
## 2016-05-01 0.022290135
## 2016-06-01 -0.056773337
## 2016-07-01 0.067913625
## 2016-08-01 0.024685852
## 2016-09-01 -0.007710034
## 2016-10-01 0.014650338
## 2016-11-01 -0.002316922
## 2016-12-01 0.079016569
## 2017-01-01 0.010273442
```

```
DAX.ret.month = monthlyReturn(DAX,type='log')[-1]
```

```
#same as for scaled prices, since  $\log(S_t / S_{t-1}) = \log((S_t/S_0) / (S_{t-1}/S_0))$ 
```

```
norm(monthlyReturn(DAX.scaled) - monthlyReturn(DAX))
```

```
## [1] 9.992007e-16
```

```
#caution! There are some small differnces (due to time scaling????)
```

```
cbind(DAX.ret.month * 12,Xi[,1])
```

```
##          monthly.returns      ..2
## 2016-02-01 -0.37658447 -0.37658447
## 2016-03-01 0.57987193 0.57987193
## 2016-04-01 0.08813265 0.08813265
## 2016-05-01 0.26454410 0.26454410
## 2016-06-01 -0.70138394 -0.70138394
## 2016-07-01 0.78848234 0.78848234
## 2016-08-01 0.29263296 0.29263296
## 2016-09-01 -0.09287892 -0.09287892
## 2016-10-01 0.17452871 0.17452871
```

```

## 2016-11-01      -0.02783533 -0.02783533
## 2016-12-01       0.91260051  0.91260051
## 2017-01-01       0.12265234  0.12265234
cbind(Xi[,1], 12*DAX.ret.month, as.numeric(time[n]-time[1])*(1/as.numeric(diff(time)))*DAX.ret.month)

##          ..1 monthly.returns monthly.returns.1
## 2016-02-01 -0.37658447      -0.37658447      -0.37051053
## 2016-03-01  0.57987193       0.57987193       0.60986530
## 2016-04-01  0.08813265       0.08813265       0.08671116
## 2016-05-01  0.26454410       0.26454410       0.26895316
## 2016-06-01 -0.70138394      -0.70138394      -0.69007130
## 2016-07-01  0.78848234       0.78848234       0.80162371
## 2016-08-01  0.29263296       0.29263296       0.28791308
## 2016-09-01 -0.09287892      -0.09287892      -0.09138087
## 2016-10-01  0.17452871       0.17452871       0.17743752
## 2016-11-01 -0.02783533      -0.02783533      -0.02738637
## 2016-12-01  0.91260051       0.91260051       0.92781051
## 2017-01-01  0.12265234       0.12265234       0.12067408
norm(Xi[,1] - 12*DAX.ret.month)

## [1] 7.921441e-14
norm(Xi[,1] - as.numeric(time[n]-time[1])*(1/as.numeric(diff(time)))*DAX.ret.month)

## [1] 0.09311587
RWE.ret.month = monthlyReturn(RWE, type = 'log')[-1]
GOLD.ret.month = monthlyReturn(GOLD, type = 'log')[-1]
OIL.ret.month = monthlyReturn(OIL, type = 'log')[-1]
USEUR.ret.month = monthlyReturn(USEUR, type = 'log')[-1]

(Xi.alternative = cbind(DAX.ret.month, RWE.ret.month, GOLD.ret.month, OIL.ret.month, USEUR.ret.month)*1)

##          monthly.returns monthly.returns.1 monthly.returns.2
## 2016-02-01      -0.37658447      -2.39665774       0.584614453
## 2016-03-01       0.57987193       0.91488506       1.261612609
## 2016-04-01       0.08813265       1.64384345      -0.079512978
## 2016-05-01       0.26454410      -1.23931076       0.568601158
## 2016-06-01      -0.70138394       2.24890163      -0.731364497
## 2016-07-01       0.78848234       1.36915241       1.028571770
## 2016-08-01       0.29263296      -0.97403787       0.218124145
## 2016-09-01      -0.09287892       0.53608914      -0.393371822
## 2016-10-01       0.17452871      -0.70501897       0.075179318
## 2016-11-01      -0.02783533      -2.37857788      -0.316690828
## 2016-12-01       0.91260051      -0.04561780      -1.043942468
## 2017-01-01       0.12265234      -0.01524455      -0.008208542
##          monthly.returns.3 monthly.returns.4
## 2016-02-01      -0.64269551      -0.04037996
## 2016-03-01       0.24772757      -0.54724785
## 2016-04-01       1.10910690      -0.07947049
## 2016-05-01       1.98694199       0.34419704
## 2016-06-01       0.56310599       0.02935292
## 2016-07-01       0.06009811      -0.07485821
## 2016-08-01      -1.75375573       0.01741947
## 2016-09-01       1.03130916      -0.08870015

```

```
## 2016-10-01      0.75108309      0.28129933
## 2016-11-01     -0.31615440      0.43603525
## 2016-12-01      0.71759804      0.08230919
## 2017-01-01      1.18890104     -0.01894438
```

```
Xi
```

```
##          DAX          RWE          gold          oil          $/\200
## [1,] -0.37658447 -2.39665774  0.584614453 -0.64269551 -0.04037996
## [2,]  0.57987193  0.91488506  1.261612609  0.24772757 -0.54724785
## [3,]  0.08813265  1.64384345 -0.079512978  1.10910690 -0.07947049
## [4,]  0.26454410 -1.23931076  0.568601158  1.98694199  0.34419704
## [5,] -0.70138394  2.24890163 -0.731364497  0.56310599  0.02935292
## [6,]  0.78848234  1.36915241  1.028571770  0.06009811 -0.07485821
## [7,]  0.29263296 -0.97403787  0.218124145 -1.75375573  0.01741947
## [8,] -0.09287892  0.53608914 -0.393371822  1.03130916 -0.08870015
## [9,]  0.17452871 -0.70501897  0.075179318  0.75108309  0.28129933
## [10,] -0.02783533 -2.37857788 -0.316690828 -0.31615440  0.43603525
## [11,]  0.91260051 -0.04561780 -1.043942468  0.71759804  0.08230919
## [12,]  0.12265234 -0.01524455 -0.008208542  1.18890104 -0.01894438
```

```
#####
```

```
#estimation of covariance matrix and expected returns
```

```
#first moment: expected annulized returns
```

```
r=colMeans(Xi)
```

```
#table 3.3 first line - mean
```

```
r*100
```

```
##          DAX          RWE          gold          oil          $/\200
## 16.873024 -8.679949  9.696769 41.193886  2.841768
```

```
#alternative computation
```

```
apply(X=Xi,MARGIN = 2,FUN = mean)
```

```
##          DAX          RWE          gold          oil          $/\200
## 0.16873024 -0.08679949  0.09696769  0.41193886  0.02841768
```

```
r
```

```
##          DAX          RWE          gold          oil          $/\200
## 0.16873024 -0.08679949  0.09696769  0.41193886  0.02841768
```

```
#second moment: variance
```

```
#unbiased variance estimator
```

```
apply(X=Xi, MARGIN = 2, FUN = var)
```

```
##          DAX          RWE          gold          oil          $/\200
## 0.20920021  2.27763288  0.46738536  0.96987541  0.06429394
```

```
#biased variance estimator (table 3.3 second line)
```

```
apply(X=Xi, MARGIN = 2, FUN = var) * (n-1)/n
```

```
##          DAX          RWE          gold          oil          $/\200
## 0.19310788  2.10243035  0.43143264  0.89526961  0.05934826
```

```
#covariance, unbiased (better estimator than biased version used in script!)
```

```
(C.unbiased = cov(Xi))
```

```
##          DAX          RWE          gold          oil          $/\200
## DAX  0.20920021  0.04480695  0.08646036  0.01445546 -0.02110378
## RWE  0.04480695  2.27763288 -0.08119001  0.48528710 -0.20649331
## gold 0.08646036 -0.08119001  0.46738536 -0.11286244 -0.07341299
## oil  0.01445546  0.48528710 -0.11286244  0.96987541  0.03142142
## $/\200 -0.02110378 -0.20649331 -0.07341299  0.03142142  0.06429394
```

```
#diagonal = variances
apply(X=Xi, MARGIN = 2, FUN = var)
```

```
##          DAX          RWE          gold          oil          $/\200
## 0.20920021 2.27763288 0.46738536 0.96987541 0.06429394
```

```
diag(C.unbiased)
```

```
##          DAX          RWE          gold          oil          $/\200
## 0.20920021 2.27763288 0.46738536 0.96987541 0.06429394
```

```
#biased covariance = version script Pichler
(C.biased = C.unbiased*(n-1)/n)
```

```
##          DAX          RWE          gold          oil          $/\200
## DAX  0.19310788  0.04136026  0.07980957  0.01334350 -0.01948041
## RWE  0.04136026  2.10243035 -0.07494462  0.44795732 -0.19060921
## gold 0.07980957 -0.07494462  0.43143264 -0.10418072 -0.06776584
## oil  0.01334350  0.44795732 -0.10418072  0.89526961  0.02900439
## $/\200 -0.01948041 -0.19060921 -0.06776584  0.02900439  0.05934826
```

```
#variances on diagonal
apply(X=Xi, MARGIN = 2, FUN = var)*(n-1)/n
```

```
##          DAX          RWE          gold          oil          $/\200
## 0.19310788 2.10243035 0.43143264 0.89526961 0.05934826
```

```
diag(C.biased)
```

```
##          DAX          RWE          gold          oil          $/\200
## 0.19310788 2.10243035 0.43143264 0.89526961 0.05934826
```

```
#####
```

```
# CAPM Markowitz: min( portfoliovar(x)) such that: expected portfolio mean(x) >= mu and sum(x)<=1
#see Theorem 3.12 (1. step: we compute the efficient portfolios for a given expected portfolio return (
# and later we write a function for doing this (step 2)
```

```
C = C.biased #because this version is used in the lecture notes
#(even using the unbiased estimator is better!!!)
```

```
#step 1:
C.inv = solve(C)
#table 3.4 (a) and 3.4 (b)
round(C*100,2)
```

```
##          DAX    RWE    gold    oil    $/\200
## DAX  19.31    4.14    7.98    1.33   -1.95
## RWE   4.14  210.24   -7.49   44.80  -19.06
## gold  7.98   -7.49   43.14  -10.42  -6.78
## oil   1.33   44.80  -10.42   89.53   2.90
## $/\200 -1.95 -19.06  -6.78   2.90   5.93
```

```

round(C.inv,2)

##          DAX   RWE   gold   oil   $\200
## DAX    5.68 -0.04 -1.00 -0.20  0.69
## RWE   -0.04  1.02  0.74 -0.57  4.38
## gold  -1.00  0.74  3.57 -0.14  6.18
## oil   -0.20 -0.57 -0.14  1.48 -2.77
## $\200  0.69  4.38  6.18 -2.77 39.56

#test
C.inv %*% C

##          DAX          RWE          gold          oil          $\200
## DAX  1.000000e+00 -1.535230e-16  7.057479e-18 -6.166400e-17  3.397957e-17
## RWE  3.970890e-18  1.000000e+00 -1.927169e-17  5.200105e-17  2.946319e-17
## gold -1.364739e-17 -1.668587e-16  1.000000e+00  6.914499e-17 -5.903481e-17
## oil  1.329503e-17  3.886865e-17  5.778798e-17  1.000000e+00 -5.045606e-17
## $\200 -6.678685e-17 -3.946496e-17 -4.152494e-16  4.836626e-16  1.000000e+00

C %*% C.inv

##          DAX          RWE          gold          oil          $\200
## DAX  1.000000e+00 -1.332891e-17 -1.364739e-17 -4.008160e-18  4.423545e-17
## RWE  1.437923e-17  1.000000e+00 -3.889033e-16  9.161508e-17 -2.704000e-15
## gold -1.835690e-17 -2.393376e-17  1.000000e+00 -2.398797e-18 -4.152494e-16
## oil  1.783851e-18 -5.502326e-18  1.363384e-17  1.000000e+00 -4.045158e-16
## $\200  6.844026e-19  8.215542e-17 -3.523657e-18 -2.550586e-17  1.000000e+00

#computation of efficient Markowitz portfolio via formula (3.5)
ones = rep(1,nr.assets)

a = as.numeric(r%*% C.inv %*% r)
b = as.numeric(r%*% C.inv %*% ones )
c = as.numeric(ones %*% C.inv %*% ones )
d = a*c-b*b

mu = seq(from=0.00,to = 0.25,by=0.01)
m = length(mu)

xm = matrix(nrow = nr.assets, ncol=m)

for (k in 1:m){
  xm[,k]= a*C.inv%*%ones- b*C.inv%*%r+ mu[k]*(c*C.inv%*%r -b*C.inv%*%ones)
}
xm = xm/d
row.names(xm) = titles

colSums(xm)

## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

xm<0

##          [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11]
## DAX FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## RWE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## gold FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE

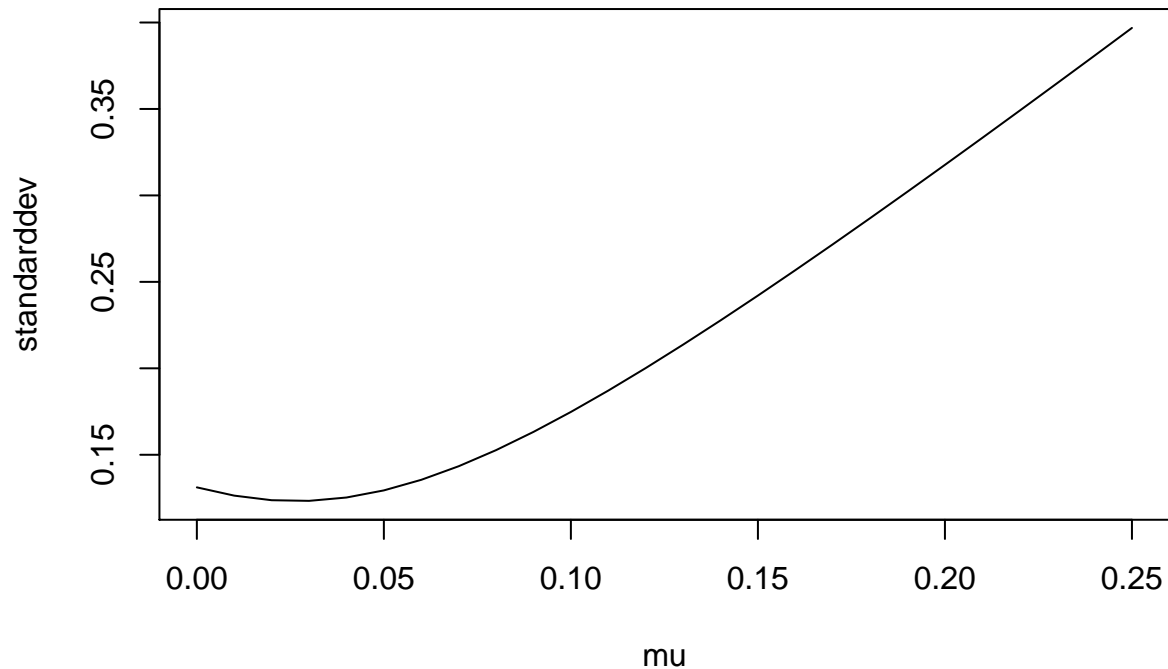
```

```
## oil TRUE TRUE TRUE TRUE TRUE FALSE FALSE FALSE FALSE FALSE FALSE
## $/\200 FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##      [,12] [,13] [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22]
## DAX FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## RWE FALSE FALSE FALSE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE
## gold FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## oil FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
## $/\200 FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##      [,23] [,24] [,25] [,26]
## DAX FALSE FALSE FALSE FALSE
## RWE TRUE TRUE TRUE TRUE
## gold FALSE FALSE FALSE FALSE
## oil FALSE FALSE FALSE FALSE
## $/\200 FALSE FALSE FALSE FALSE
```

```
standarddev = sqrt((mu^2*c-2*mu*b+a)/d)
```

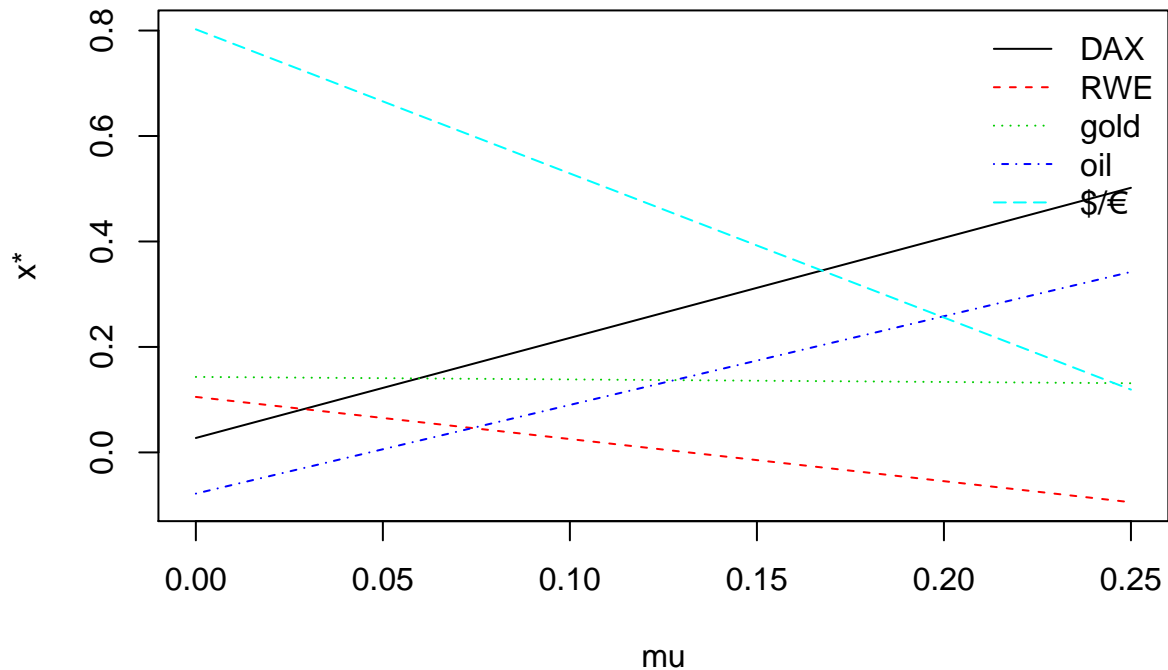
```
plot(mu,standarddev,type="l",main="Standard deviation of the optimal portfolio in dependence of mu")
```

## Standard deviation of the optimal portfolio in dependence of mu



```
matplot(mu,t(xm),type='l',col=1:nr.assets,lty=1:nr.assets,ylab="x*",main="asset allocation")
legend("topright", titles, col=1:nr.assets, lty=1:nr.assets,bty="n",lwd=1)
```

## asset allocation



```
#####
# function for creating asset allocation plot (see script figure 3.4 and 3.6)
```

```
graph.asset.allocation = function(portfolio,sd,mu, png.name="optimal-asset-allocation-mu.png",title="Ma
  nr = dim(portfolio)[1]
  titles = row.names(portfolio)
  print(portfolio)
  print(titles)

  par(mar=c(5,4,4,5)+.1)
  plot(mu,t(portfolio[1,]),ylim=c(min(portfolio),max(portfolio)),type='l',col=1,lty=1,ylab="x* asset al
  for (j in 2:nr) lines(mu,t(portfolio[j,]),type='l',col=j,lty=j)
  par(new=TRUE)
  plot(mu, sd,type="l",col="darkgray",lwd=3,lty=6,xaxt="n",yaxt="n",xlab="",ylab="")
  axis(4)
  mtext("sd(x*(mu))",side=4,line=3,col="darkgray")
  legend("topleft", c(titles), col=c(1:nr), lty=1:nr,bty="n",lwd=rep(1,nr))

  dev.copy(png,paste("pics-out/",png.name,sep=""))
  dev.off()
}
```

```
graph.asset.allocation(portfolio=xm,sd=standarddev, mu=mu,png.name = 'test.png')
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]
## DAX      0.02739916  0.04637582  0.06535247  0.08432912  0.10330578
```



```

## RWE 0.10525296 0.09726392 0.08927488 0.08128584 0.07329680
## gold 0.14319142 0.14271236 0.14223330 0.14175424 0.14127518
## oil -0.07809465 -0.06127758 -0.04446051 -0.02764344 -0.01082637
## $/\200 0.80225111 0.77492548 0.74759986 0.72027424 0.69294861
##      [,6]      [,7]      [,8]      [,9]     [,10]     [,11]
## DAX 0.122282431 0.14125908 0.16023574 0.17921239 0.19818905 0.21716570
## RWE 0.065307762 0.05731872 0.04932968 0.04134064 0.03335160 0.02536256
## gold 0.140796125 0.14031707 0.13983801 0.13935895 0.13887989 0.13840083
## oil 0.005990696 0.02280776 0.03962483 0.05644190 0.07325897 0.09007604
## $/\200 0.665622987 0.63829736 0.61097174 0.58364611 0.55632049 0.52899487
##      [,12]     [,13]     [,14]     [,15]     [,16]
## DAX 0.23614235 0.255119007 0.274095661 0.293072314 0.31204897
## RWE 0.01737353 0.009384486 0.001395446 -0.006593593 -0.01458263
## gold 0.13792177 0.137442711 0.136963652 0.136484593 0.13600553
## oil 0.10689311 0.123710180 0.140527249 0.157344318 0.17416139
## $/\200 0.50166924 0.474343617 0.447017993 0.419692369 0.39236674
##      [,17]     [,18]     [,19]     [,20]     [,21]
## DAX 0.33102562 0.35000228 0.36897893 0.38795558 0.40693224
## RWE -0.02257167 -0.03056071 -0.03854975 -0.04653879 -0.05452783
## gold 0.13552647 0.13504742 0.13456836 0.13408930 0.13361024
## oil 0.19097846 0.20779553 0.22461259 0.24142966 0.25824673
## $/\200 0.36504112 0.33771550 0.31038987 0.28306425 0.25573862
##      [,22]     [,23]     [,24]     [,25]     [,26]
## DAX 0.42590889 0.44488554 0.46386220 0.48283885 0.50181551
## RWE -0.06251687 -0.07050591 -0.07849495 -0.08648399 -0.09447303
## gold 0.13313118 0.13265212 0.13217306 0.13169400 0.13121494
## oil 0.27506380 0.29188087 0.30869794 0.32551501 0.34233208
## $/\200 0.22841300 0.20108737 0.17376175 0.14643613 0.11911050
## [1] "DAX" "RWE" "gold" "oil" "$/\200"

## pdf
## 2

```

```
#####
```

```
#step 2: function for efficient Markowitz portfolio (Theorem 3.12)
```

```

markowitz.portfolio = function(mu.ret, Cov.Matrix=diag(length(mu.ret)), mu.portfolio.min = 0){
  if (missing(mu.ret)) stop("need vector of expected asset returns: mu.ret")
  nr = length(mu.ret)
  titles= names(mu.ret)
  if (sum(dim(Cov.Matrix)==nr)<2) stop("wrong dimensions")
  ones = rep(1,nr)
  Cov.inv = solve(Cov.Matrix)
  m = length(mu.portfolio.min)
  xm = matrix(nrow = nr, ncol=m)

  a = as.numeric(mu.ret%% Cov.inv %% mu.ret)
  b = as.numeric(mu.ret%% Cov.inv %% ones )
  c = as.numeric(ones %% Cov.inv %% ones )
  d = a*c-b*b

  for (k in 1:m){
    xm[,k] = a*Cov.inv%%ones- b*Cov.inv%%mu.ret+ mu.portfolio.min[k] *(c*Cov.inv%%mu.ret -b*Cov.inv%
  }
}

```

```

if (length(titles)==0) titles=paste(rep("asset",nr),1:nr)
#print(titles)
row.names(xm) = titles
standarddev = sqrt((mu.portfolio.min^2*c-2*mu.portfolio.min*b+a)/d)
return.list = list(xm/d,standarddev)
names(return.list) = c("efficient_portfolio","standarddev")
return(return.list)
}

```

```
(test = markowitz.portfolio(r,C,0.03))
```

```

## $efficient_portfolio
##           [,1]
## DAX      0.08432912
## RWE      0.08128584
## gold     0.14175424
## oil     -0.02764344
## $/\200   0.72027424
##
## $standarddev
## [1] 0.1233607

```

```
sum(test$efficient_portfolio)
```

```
## [1] 1
```

```
(test2 = markowitz.portfolio(r,C,mu))
```

```

## $efficient_portfolio
##           [,1]      [,2]      [,3]      [,4]      [,5]
## DAX      0.02739916  0.04637582  0.06535247  0.08432912  0.10330578
## RWE      0.10525296  0.09726392  0.08927488  0.08128584  0.07329680
## gold     0.14319142  0.14271236  0.14223330  0.14175424  0.14127518
## oil     -0.07809465 -0.06127758 -0.04446051 -0.02764344 -0.01082637
## $/\200   0.80225111  0.77492548  0.74759986  0.72027424  0.69294861
##           [,6]      [,7]      [,8]      [,9]      [,10]      [,11]
## DAX      0.122282431  0.14125908  0.16023574  0.17921239  0.19818905  0.21716570
## RWE      0.065307762  0.05731872  0.04932968  0.04134064  0.03335160  0.02536256
## gold     0.140796125  0.14031707  0.13983801  0.13935895  0.13887989  0.13840083
## oil      0.005990696  0.02280776  0.03962483  0.05644190  0.07325897  0.09007604
## $/\200   0.665622987  0.63829736  0.61097174  0.58364611  0.55632049  0.52899487
##           [,12]      [,13]      [,14]      [,15]      [,16]
## DAX      0.23614235  0.255119007  0.274095661  0.293072314  0.31204897
## RWE      0.01737353  0.009384486  0.001395446 -0.006593593 -0.01458263
## gold     0.13792177  0.137442711  0.136963652  0.136484593  0.13600553
## oil      0.10689311  0.123710180  0.140527249  0.157344318  0.17416139
## $/\200   0.50166924  0.474343617  0.447017993  0.419692369  0.39236674
##           [,17]      [,18]      [,19]      [,20]      [,21]
## DAX      0.33102562  0.35000228  0.36897893  0.38795558  0.40693224
## RWE     -0.02257167 -0.03056071 -0.03854975 -0.04653879 -0.05452783
## gold     0.13552647  0.13504742  0.13456836  0.13408930  0.13361024
## oil      0.19097846  0.20779553  0.22461259  0.24142966  0.25824673
## $/\200   0.36504112  0.33771550  0.31038987  0.28306425  0.25573862
##           [,22]      [,23]      [,24]      [,25]      [,26]

```

```
## DAX 0.42590889 0.44488554 0.46386220 0.48283885 0.50181551
## RWE -0.06251687 -0.07050591 -0.07849495 -0.08648399 -0.09447303
## gold 0.13313118 0.13265212 0.13217306 0.13169400 0.13121494
## oil 0.27506380 0.29188087 0.30869794 0.32551501 0.34233208
## $\200 0.22841300 0.20108737 0.17376175 0.14643613 0.11911050
##
## $standarddev
## [1] 0.1311631 0.1263784 0.1237317 0.1233607 0.1252856 0.1294040 0.1355160
## [8] 0.1433670 0.1526889 0.1632299 0.1747695 0.1871231 0.2001400 0.2136991
## [15] 0.2277035 0.2420759 0.2567545 0.2716898 0.2868416 0.3021773 0.3176704
## [22] 0.3332988 0.3490444 0.3648921 0.3808290 0.3968445
```

```
colSums(test2$efficient_portfolio,)
```

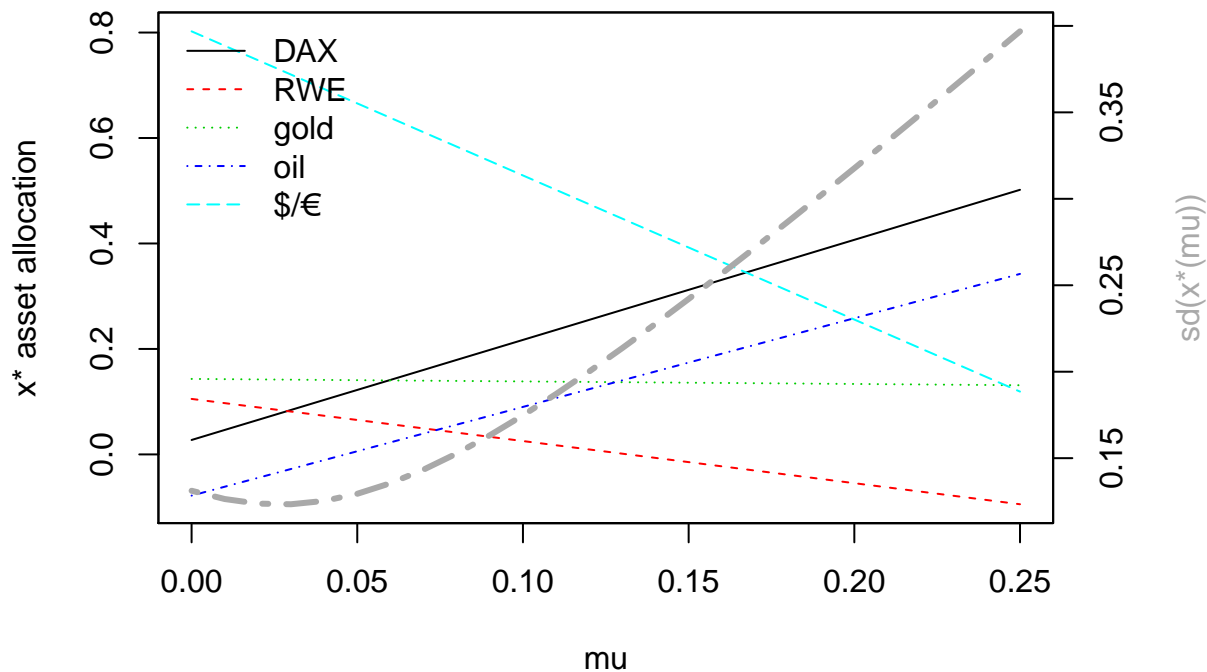
```
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

```
row.names(test2$efficient_portfolio)
```

```
## [1] "DAX" "RWE" "gold" "oil" "$\200"
```

```
graph.asset.allocation(portfolio=test2$efficient_portfolio,sd=test2$standarddev, mu=mu, png.name = 'ass3')
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]
## DAX 0.02739916 0.04637582 0.06535247 0.08432912 0.10330578
## RWE 0.10525296 0.09726392 0.08927488 0.08128584 0.07329680
## gold 0.14319142 0.14271236 0.14223330 0.14175424 0.14127518
## oil -0.07809465 -0.06127758 -0.04446051 -0.02764344 -0.01082637
## $\200 0.80225111 0.77492548 0.74759986 0.72027424 0.69294861
##           [,6]      [,7]      [,8]      [,9]      [,10]      [,11]
## DAX 0.122282431 0.14125908 0.16023574 0.17921239 0.19818905 0.21716570
## RWE 0.065307762 0.05731872 0.04932968 0.04134064 0.03335160 0.02536256
## gold 0.140796125 0.14031707 0.13983801 0.13935895 0.13887989 0.13840083
## oil 0.005990696 0.02280776 0.03962483 0.05644190 0.07325897 0.09007604
## $\200 0.665622987 0.63829736 0.61097174 0.58364611 0.55632049 0.52899487
##           [,12]      [,13]      [,14]      [,15]      [,16]
## DAX 0.23614235 0.255119007 0.274095661 0.293072314 0.31204897
## RWE 0.01737353 0.009384486 0.001395446 -0.006593593 -0.01458263
## gold 0.13792177 0.137442711 0.136963652 0.136484593 0.13600553
## oil 0.10689311 0.123710180 0.140527249 0.157344318 0.17416139
## $\200 0.50166924 0.474343617 0.447017993 0.419692369 0.39236674
##           [,17]      [,18]      [,19]      [,20]      [,21]
## DAX 0.33102562 0.35000228 0.36897893 0.38795558 0.40693224
## RWE -0.02257167 -0.03056071 -0.03854975 -0.04653879 -0.05452783
## gold 0.13552647 0.13504742 0.13456836 0.13408930 0.13361024
## oil 0.19097846 0.20779553 0.22461259 0.24142966 0.25824673
## $\200 0.36504112 0.33771550 0.31038987 0.28306425 0.25573862
##           [,22]      [,23]      [,24]      [,25]      [,26]
## DAX 0.42590889 0.44488554 0.46386220 0.48283885 0.50181551
## RWE -0.06251687 -0.07050591 -0.07849495 -0.08648399 -0.09447303
## gold 0.13313118 0.13265212 0.13217306 0.13169400 0.13121494
## oil 0.27506380 0.29188087 0.30869794 0.32551501 0.34233208
## $\200 0.22841300 0.20108737 0.17376175 0.14643613 0.11911050
## [1] "DAX" "RWE" "gold" "oil" "$\200"
```



```
## pdf
## 2
```

```
#####
```

```
#theorem 3.19 - Markowitz with risk free asset - section 3.8, Theorem 3.19
```

```
markowitz.portfolio.cash = function(mu.ret, Cov.Matrix=diag(length(mu.ret)), mu.portfolio.min = 0, r0=0)
  if (missing(mu.ret)) stop("need vector of expected asset returns: mu.ret")
  titles= names(mu.ret)
```

```
nr = length(mu.ret)
if (sum(dim(Cov.Matrix)==nr)<2) stop("wrong dimensions")
ones = rep(1,nr)
Cov.inv = solve(Cov.Matrix)
```

```
m = length(mu.portfolio.min)
xm.r0 = matrix(nrow = nr+1, ncol=m)
#first nr compontens usual assets x^star, nr+1 = cash(r0) x0^star
```

```
a = as.numeric(mu.ret%% Cov.inv %% mu.ret)
b = as.numeric(mu.ret%% Cov.inv %% ones )
c = as.numeric(ones %% Cov.inv %% ones )
d = a-2*b*r0+c*r0*r0
```

```
for (k in 1:m){
  xm.r0[,k] = c((mu.portfolio.min[k]-r0)*(Cov.inv%%mu.ret - r0 * Cov.inv%%ones),d-(b-r0*c)*(mu.port
```

```

}

if (length(assets)==0) titles=paste(rep("asset",nr),1:nr)
print(titles)
row.names(xm.r0) = c(titles,"cash")
standarddev = sqrt(((mu.portfolio.min-r0)^2/d))
return.list = list(xm.r0/d,standarddev)
names(return.list) = c("efficient_portfolio","standarddev")
return(return.list)
}

r0 = 0.02

mu.new = seq(from=0.0, by=0.01, to=0.84)

(portfolio.riskfree = markowitz.portfolio.cash(r,C,mu.new,r0))

## [1] "DAX" "RWE" "gold" "oil" "$/\200"

## $efficient_portfolio
##           [,1]      [,2] [,3]      [,4]      [,5]      [,6]
## DAX -0.03956187 -0.019780937  0  0.019780937  0.03956187  0.059342810
## RWE  0.01378069  0.006890347  0 -0.006890347 -0.01378069 -0.020671041
## gold -0.00254277 -0.001271385  0  0.001271385  0.00254277  0.003814154
## oil  -0.03253980 -0.016269900  0  0.016269900  0.03253980  0.048809701
## $/\200  0.03625005  0.018125025  0 -0.018125025 -0.03625005 -0.054375076
## cash  1.02461370  1.012306850  1  0.987693150  0.97538630  0.963079451
##           [,7]      [,8]      [,9]      [,10]     [,11]
## DAX  0.079123747  0.098904684  0.118685620  0.138466557  0.15824749
## RWE -0.027561388 -0.034451735 -0.041342082 -0.048232429 -0.05512278
## gold  0.005085539  0.006356924  0.007628309  0.008899693  0.01017108
## oil  0.065079601  0.081349502  0.097619402  0.113889302  0.13015920
## $/\200 -0.072500101 -0.090625126 -0.108750151 -0.126875176 -0.14500020
## cash  0.950772601  0.938465752  0.926158902  0.913852052  0.90154520
##           [,12]     [,13]     [,14]     [,15]     [,16]
## DAX  0.17802843  0.19780937  0.21759030  0.23737124  0.25715218
## RWE -0.06201312 -0.06890347 -0.07579382 -0.08268416 -0.08957451
## gold  0.01144246  0.01271385  0.01398523  0.01525662  0.01652800
## oil  0.14642910  0.16269900  0.17896890  0.19523880  0.21150870
## $/\200 -0.16312523 -0.18125025 -0.19937528 -0.21750030 -0.23562533
## cash  0.88923835  0.87693150  0.86462465  0.85231780  0.84001095
##           [,17]     [,18]     [,19]     [,20]     [,21]
## DAX  0.27693311  0.29671405  0.31649499  0.33627592  0.35605686
## RWE -0.09646486 -0.10335520 -0.11024555 -0.11713590 -0.12402625
## gold  0.01779939  0.01907077  0.02034216  0.02161354  0.02288493
## oil  0.22777860  0.24404850  0.26031841  0.27658831  0.29285821
## $/\200 -0.25375035 -0.27187538 -0.29000040 -0.30812543 -0.32625045
## cash  0.82770410  0.81539725  0.80309041  0.79078356  0.77847671
##           [,22]     [,23]     [,24]     [,25]     [,26]
## DAX  0.37583780  0.3956187  0.41539967  0.43518061  0.45496155
## RWE -0.13091659 -0.1378069 -0.14469729 -0.15158763 -0.15847798
## gold  0.02415631  0.0254277  0.02669908  0.02797047  0.02924185
## oil  0.30912811  0.3253980  0.34166791  0.35793781  0.37420771
## $/\200 -0.34437548 -0.3625005 -0.38062553 -0.39875055 -0.41687558

```

```

## cash 0.76616986 0.7538630 0.74155616 0.72924931 0.71694246
##      [,27]      [,28]      [,29]      [,30]      [,31]
## DAX 0.47474248 0.49452342 0.5143044 0.53408529 0.55386623
## RWE -0.16536833 -0.17225867 -0.1791490 -0.18603937 -0.19292971
## gold 0.03051323 0.03178462 0.0330560 0.03432739 0.03559877
## oil 0.39047761 0.40674751 0.4230174 0.43928731 0.45555721
## $/\200 -0.43500060 -0.45312563 -0.4712507 -0.48937568 -0.50750071
## cash 0.70463561 0.69232876 0.6800219 0.66771506 0.65540821
##      [,32]      [,33]      [,34]      [,35]      [,36]
## DAX 0.57364717 0.59342810 0.61320904 0.63298998 0.6527709
## RWE -0.19982006 -0.20671041 -0.21360076 -0.22049110 -0.2273814
## gold 0.03687016 0.03814154 0.03941293 0.04068431 0.0419557
## oil 0.47182711 0.48809701 0.50436691 0.52063681 0.5369067
## $/\200 -0.52562573 -0.54375076 -0.56187578 -0.58000081 -0.5981258
## cash 0.64310136 0.63079451 0.61848766 0.60618081 0.5938740
##      [,37]      [,38]      [,39]      [,40]      [,41]
## DAX 0.67255185 0.69233279 0.71211372 0.73189466 0.75167560
## RWE -0.23427180 -0.24116214 -0.24805249 -0.25494284 -0.26183318
## gold 0.04322708 0.04449847 0.04576985 0.04704124 0.04831262
## oil 0.55317661 0.56944651 0.58571641 0.60198631 0.61825621
## $/\200 -0.61625086 -0.63437588 -0.65250091 -0.67062593 -0.68875096
## cash 0.58156711 0.56926026 0.55695341 0.54464656 0.53233971
##      [,42]      [,43]      [,44]      [,45]      [,46]
## DAX 0.77145653 0.79123747 0.81101841 0.83079934 0.85058028
## RWE -0.26872353 -0.27561388 -0.28250423 -0.28939457 -0.29628492
## gold 0.04958401 0.05085539 0.05212678 0.05339816 0.05466955
## oil 0.63452611 0.65079601 0.66706591 0.68333581 0.69960571
## $/\200 -0.70687598 -0.72500101 -0.74312603 -0.76125106 -0.77937608
## cash 0.52003286 0.50772601 0.49541916 0.48311231 0.47080546
##      [,47]      [,48]      [,49]      [,50]      [,51]
## DAX 0.87036122 0.89014215 0.9099231 0.92970403 0.94948496
## RWE -0.30317527 -0.31006561 -0.3169560 -0.32384631 -0.33073665
## gold 0.05594093 0.05721231 0.0584837 0.05975508 0.06102647
## oil 0.71587561 0.73214551 0.7484154 0.76468531 0.78095522
## $/\200 -0.79750111 -0.81562613 -0.8337512 -0.85187618 -0.87000121
## cash 0.45849861 0.44619176 0.4338849 0.42157807 0.40927122
##      [,52]      [,53]      [,54]      [,55]      [,56]
## DAX 0.96926590 0.98904684 1.00882777 1.02860871 1.04838965
## RWE -0.33762700 -0.34451735 -0.35140769 -0.35829804 -0.36518839
## gold 0.06229785 0.06356924 0.06484062 0.06611201 0.06738339
## oil 0.79722512 0.81349502 0.82976492 0.84603482 0.86230472
## $/\200 -0.88812623 -0.90625126 -0.92437629 -0.94250131 -0.96062634
## cash 0.39696437 0.38465752 0.37235067 0.36004382 0.34773697
##      [,57]      [,58]      [,59]      [,60]      [,61]
## DAX 1.06817058 1.08795152 1.10773246 1.12751339 1.14729433
## RWE -0.37207874 -0.37896908 -0.38585943 -0.39274978 -0.39964012
## gold 0.06865478 0.06992616 0.07119755 0.07246893 0.07374032
## oil 0.87857462 0.89484452 0.91111442 0.92738432 0.94365422
## $/\200 -0.97875136 -0.99687639 -1.01500141 -1.03312644 -1.05125146
## cash 0.33543012 0.32312327 0.31081642 0.29850957 0.28620272
##      [,62]      [,63]      [,64]      [,65]      [,66]
## DAX 1.1670753 1.18685620 1.20663714 1.22641808 1.24619901
## RWE -0.4065305 -0.41342082 -0.42031116 -0.42720151 -0.43409186
## gold 0.0750117 0.07628309 0.07755447 0.07882586 0.08009724

```



```
graph.asset.allocation(portfolio=portfolio.riskfree$efficient_portfolio,sd=portfolio.riskfree$standardd
mu=mu.new, png.name = 'asset-allocation-with-cash.png')
```

```
##           [,1]           [,2] [,3]           [,4]           [,5]           [,6]
## DAX -0.03956187 -0.019780937  0  0.019780937  0.03956187  0.059342810
## RWE  0.01378069  0.006890347  0 -0.006890347 -0.01378069 -0.020671041
## gold -0.00254277 -0.001271385  0  0.001271385  0.00254277  0.003814154
## oil -0.03253980 -0.016269900  0  0.016269900  0.03253980  0.048809701
## $/\200  0.03625005  0.018125025  0 -0.018125025 -0.03625005 -0.054375076
## cash  1.02461370  1.012306850  1  0.987693150  0.97538630  0.963079451
##           [,7]           [,8]           [,9]           [,10]          [,11]
## DAX  0.079123747  0.098904684  0.118685620  0.138466557  0.15824749
## RWE -0.027561388 -0.034451735 -0.041342082 -0.048232429 -0.05512278
## gold  0.005085539  0.006356924  0.007628309  0.008899693  0.010171108
## oil  0.065079601  0.081349502  0.097619402  0.113889302  0.13015920
## $/\200 -0.072500101 -0.090625126 -0.108750151 -0.126875176 -0.14500020
## cash  0.950772601  0.938465752  0.926158902  0.913852052  0.90154520
##           [,12]          [,13]          [,14]          [,15]          [,16]
## DAX  0.17802843  0.19780937  0.21759030  0.23737124  0.25715218
## RWE -0.06201312 -0.06890347 -0.07579382 -0.08268416 -0.08957451
## gold  0.01144246  0.01271385  0.01398523  0.01525662  0.01652800
## oil  0.14642910  0.16269900  0.17896890  0.19523880  0.21150870
## $/\200 -0.16312523 -0.18125025 -0.19937528 -0.21750030 -0.23562533
## cash  0.88923835  0.87693150  0.86462465  0.85231780  0.84001095
##           [,17]          [,18]          [,19]          [,20]          [,21]
## DAX  0.27693311  0.29671405  0.31649499  0.33627592  0.35605686
## RWE -0.09646486 -0.10335520 -0.11024555 -0.11713590 -0.12402625
## gold  0.01779939  0.01907077  0.02034216  0.02161354  0.02288493
## oil  0.22777860  0.24404850  0.26031841  0.27658831  0.29285821
## $/\200 -0.25375035 -0.27187538 -0.29000040 -0.30812543 -0.32625045
## cash  0.82770410  0.81539725  0.80309041  0.79078356  0.77847671
##           [,22]          [,23]          [,24]          [,25]          [,26]
## DAX  0.37583780  0.3956187  0.41539967  0.43518061  0.45496155
## RWE -0.13091659 -0.1378069 -0.14469729 -0.15158763 -0.15847798
## gold  0.02415631  0.0254277  0.02669908  0.02797047  0.02924185
## oil  0.30912811  0.3253980  0.34166791  0.35793781  0.37420771
## $/\200 -0.34437548 -0.3625005 -0.38062553 -0.39875055 -0.41687558
## cash  0.76616986  0.7538630  0.74155616  0.72924931  0.71694246
##           [,27]          [,28]          [,29]          [,30]          [,31]
## DAX  0.47474248  0.49452342  0.5143044  0.53408529  0.55386623
## RWE -0.16536833 -0.17225867 -0.1791490 -0.18603937 -0.19292971
## gold  0.03051323  0.03178462  0.0330560  0.03432739  0.03559877
## oil  0.39047761  0.40674751  0.4230174  0.43928731  0.45555721
## $/\200 -0.43500060 -0.45312563 -0.4712507 -0.48937568 -0.50750071
## cash  0.70463561  0.69232876  0.6800219  0.66771506  0.65540821
##           [,32]          [,33]          [,34]          [,35]          [,36]
## DAX  0.57364717  0.59342810  0.61320904  0.63298998  0.6527709
## RWE -0.19982006 -0.20671041 -0.21360076 -0.22049110 -0.2273814
## gold  0.03687016  0.03814154  0.03941293  0.04068431  0.0419557
## oil  0.47182711  0.48809701  0.50436691  0.52063681  0.5369067
## $/\200 -0.52562573 -0.54375076 -0.56187578 -0.58000081 -0.5981258
## cash  0.64310136  0.63079451  0.61848766  0.60618081  0.5938740
##           [,37]          [,38]          [,39]          [,40]          [,41]
## DAX  0.67255185  0.69233279  0.71211372  0.73189466  0.75167560
```

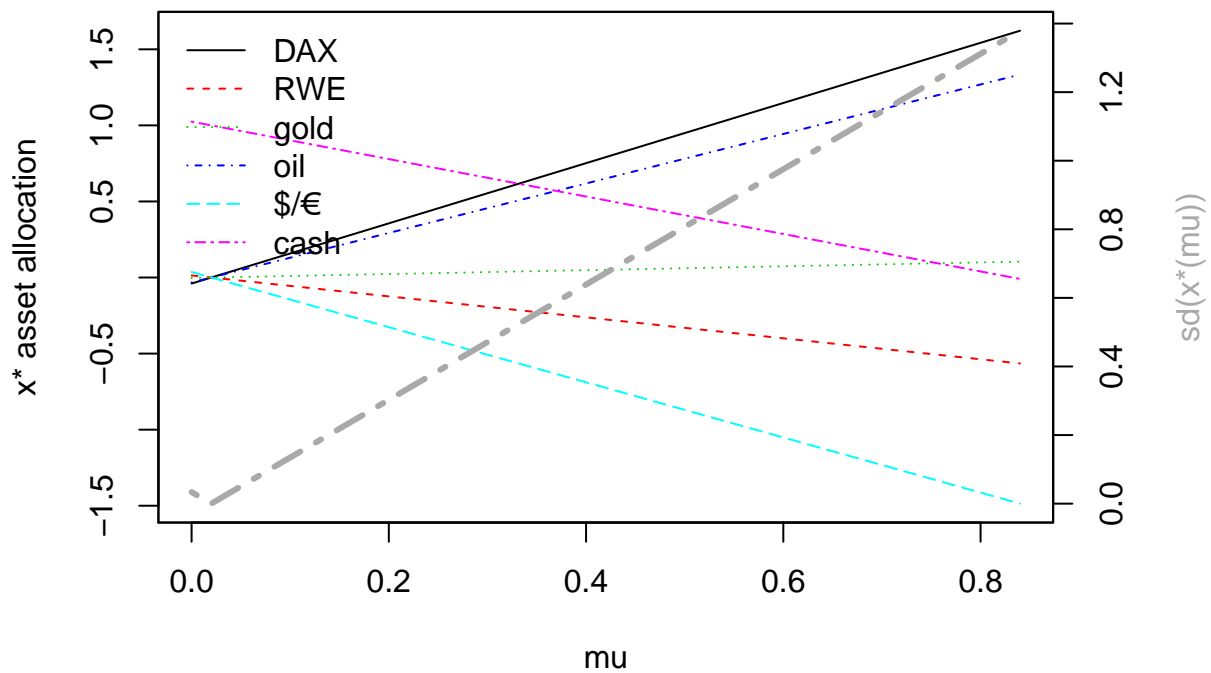


```

## RWE -0.23427180 -0.24116214 -0.24805249 -0.25494284 -0.26183318
## gold 0.04322708 0.04449847 0.04576985 0.04704124 0.04831262
## oil 0.55317661 0.56944651 0.58571641 0.60198631 0.61825621
## $\200 -0.61625086 -0.63437588 -0.65250091 -0.67062593 -0.68875096
## cash 0.58156711 0.56926026 0.55695341 0.54464656 0.53233971
##      [,42]      [,43]      [,44]      [,45]      [,46]
## DAX 0.77145653 0.79123747 0.81101841 0.83079934 0.85058028
## RWE -0.26872353 -0.27561388 -0.28250423 -0.28939457 -0.29628492
## gold 0.04958401 0.05085539 0.05212678 0.05339816 0.05466955
## oil 0.63452611 0.65079601 0.66706591 0.68333581 0.69960571
## $\200 -0.70687598 -0.72500101 -0.74312603 -0.76125106 -0.77937608
## cash 0.52003286 0.50772601 0.49541916 0.48311231 0.47080546
##      [,47]      [,48]      [,49]      [,50]      [,51]
## DAX 0.87036122 0.89014215 0.9099231 0.92970403 0.94948496
## RWE -0.30317527 -0.31006561 -0.3169560 -0.32384631 -0.33073665
## gold 0.05594093 0.05721231 0.0584837 0.05975508 0.06102647
## oil 0.71587561 0.73214551 0.7484154 0.76468531 0.78095522
## $\200 -0.79750111 -0.81562613 -0.8337512 -0.85187618 -0.87000121
## cash 0.45849861 0.44619176 0.4338849 0.42157807 0.40927122
##      [,52]      [,53]      [,54]      [,55]      [,56]
## DAX 0.96926590 0.98904684 1.00882777 1.02860871 1.04838965
## RWE -0.33762700 -0.34451735 -0.35140769 -0.35829804 -0.36518839
## gold 0.06229785 0.06356924 0.06484062 0.06611201 0.06738339
## oil 0.79722512 0.81349502 0.82976492 0.84603482 0.86230472
## $\200 -0.88812623 -0.90625126 -0.92437629 -0.94250131 -0.96062634
## cash 0.39696437 0.38465752 0.37235067 0.36004382 0.34773697
##      [,57]      [,58]      [,59]      [,60]      [,61]
## DAX 1.06817058 1.08795152 1.10773246 1.12751339 1.14729433
## RWE -0.37207874 -0.37896908 -0.38585943 -0.39274978 -0.39964012
## gold 0.06865478 0.06992616 0.07119755 0.07246893 0.07374032
## oil 0.87857462 0.89484452 0.91111442 0.92738432 0.94365422
## $\200 -0.97875136 -0.99687639 -1.01500141 -1.03312644 -1.05125146
## cash 0.33543012 0.32312327 0.31081642 0.29850957 0.28620272
##      [,62]      [,63]      [,64]      [,65]      [,66]
## DAX 1.1670753 1.18685620 1.20663714 1.22641808 1.24619901
## RWE -0.4065305 -0.41342082 -0.42031116 -0.42720151 -0.43409186
## gold 0.0750117 0.07628309 0.07755447 0.07882586 0.08009724
## oil 0.9599241 0.97619402 0.99246392 1.00873382 1.02500372
## $\200 -1.0693765 -1.08750151 -1.10562654 -1.12375156 -1.14187659
## cash 0.2738959 0.26158902 0.24928217 0.23697532 0.22466847
##      [,67]      [,68]      [,69]      [,70]      [,71]
## DAX 1.26597995 1.28576089 1.3055418 1.32532276 1.34510370
## RWE -0.44098220 -0.44787255 -0.4547629 -0.46165325 -0.46854359
## gold 0.08136863 0.08264001 0.0839114 0.08518278 0.08645416
## oil 1.04127362 1.05754352 1.0738134 1.09008332 1.10635322
## $\200 -1.16000161 -1.17812664 -1.1962517 -1.21437669 -1.23250171
## cash 0.21236162 0.20005477 0.1877479 0.17544107 0.16313422
##      [,72]      [,73]      [,74]      [,75]      [,76]
## DAX 1.36488464 1.38466557 1.40444651 1.4242274 1.44400838
## RWE -0.47543394 -0.48232429 -0.48921463 -0.4961050 -0.50299533
## gold 0.08772555 0.08899693 0.09026832 0.0915397 0.09281109
## oil 1.12262312 1.13889302 1.15516292 1.1714328 1.18770272
## $\200 -1.25062674 -1.26875176 -1.28687679 -1.3050018 -1.32312684
## cash 0.15082737 0.13852052 0.12621367 0.1139068 0.10159997

```

```
##          [,77]      [,78]      [,79]      [,80]      [,81]
## DAX    1.46378932  1.48357026  1.50335119  1.52313213  1.54291307
## RWE   -0.50988567 -0.51677602 -0.52366637 -0.53055672 -0.53744706
## gold   0.09408247  0.09535386  0.09662524  0.09789663  0.09916801
## oil    1.20397262  1.22024252  1.23651242  1.25278232  1.26905222
## $/\200 -1.34125186 -1.35937689 -1.37750192 -1.39562694 -1.41375197
## cash   0.08929312  0.07698627  0.06467942  0.05237257  0.04006573
##          [,82]      [,83]      [,84]      [,85]
## DAX    1.56269400  1.58247494  1.602255876  1.622036813
## RWE   -0.54433741 -0.55122776 -0.558118103 -0.565008450
## gold   0.10043940  0.10171078  0.102982167  0.104253552
## oil    1.28532212  1.30159203  1.317861926  1.334131826
## $/\200 -1.43187699 -1.45000202 -1.468127041 -1.486252067
## cash   0.02775888  0.01545203  0.003145176 -0.009161674
## [1] "DAX" "RWE" "gold" "oil" "$/\200" "cash"
```



```
## pdf
## 2
#
# -> all functions exported to markowitz-functions.r -> including with source("markowitz-functions.r")
#### same with unbiased covariance matrix

(mw.portfolio = markowitz.portfolio(r,C.unbiased,mu))

## $efficient_portfolio
##          [,1]      [,2]      [,3]      [,4]      [,5]
```

```

## DAX 0.02739916 0.04637582 0.06535247 0.08432912 0.10330578
## RWE 0.10525296 0.09726392 0.08927488 0.08128584 0.07329680
## gold 0.14319142 0.14271236 0.14223330 0.14175424 0.14127518
## oil -0.07809465 -0.06127758 -0.04446051 -0.02764344 -0.01082637
## $/\200 0.80225111 0.77492548 0.74759986 0.72027424 0.69294861
##      [,6]      [,7]      [,8]      [,9]     [,10]     [,11]
## DAX 0.122282431 0.14125908 0.16023574 0.17921239 0.19818905 0.21716570
## RWE 0.065307762 0.05731872 0.04932968 0.04134064 0.03335160 0.02536256
## gold 0.140796125 0.14031707 0.13983801 0.13935895 0.13887989 0.13840083
## oil 0.005990696 0.02280776 0.03962483 0.05644190 0.07325897 0.09007604
## $/\200 0.665622987 0.63829736 0.61097174 0.58364611 0.55632049 0.52899487
##      [,12]     [,13]     [,14]     [,15]     [,16]
## DAX 0.23614235 0.255119007 0.274095661 0.293072314 0.31204897
## RWE 0.01737353 0.009384486 0.001395446 -0.006593593 -0.01458263
## gold 0.13792177 0.137442711 0.136963652 0.136484593 0.13600553
## oil 0.10689311 0.123710180 0.140527249 0.157344318 0.17416139
## $/\200 0.50166924 0.474343617 0.447017993 0.419692369 0.39236674
##      [,17]     [,18]     [,19]     [,20]     [,21]
## DAX 0.33102562 0.35000228 0.36897893 0.38795558 0.40693224
## RWE -0.02257167 -0.03056071 -0.03854975 -0.04653879 -0.05452783
## gold 0.13552647 0.13504742 0.13456836 0.13408930 0.13361024
## oil 0.19097846 0.20779553 0.22461259 0.24142966 0.25824673
## $/\200 0.36504112 0.33771550 0.31038987 0.28306425 0.25573862
##      [,22]     [,23]     [,24]     [,25]     [,26]
## DAX 0.42590889 0.44488554 0.46386220 0.48283885 0.50181551
## RWE -0.06251687 -0.07050591 -0.07849495 -0.08648399 -0.09447303
## gold 0.13313118 0.13265212 0.13217306 0.13169400 0.13121494
## oil 0.27506380 0.29188087 0.30869794 0.32551501 0.34233208
## $/\200 0.22841300 0.20108737 0.17376175 0.14643613 0.11911050
##
## $standarddev
## [1] 0.1365189 0.1315388 0.1287840 0.1283979 0.1304013 0.1346879 0.1410495
## [8] 0.1492211 0.1589236 0.1698950 0.1819058 0.1947639 0.2083123 0.2224251
## [15] 0.2370013 0.2519606 0.2672386 0.2827837 0.2985542 0.3145161 0.3306418
## [22] 0.3469084 0.3632970 0.3797917 0.3963794 0.4130489

```

```
colSums(mw.portfolio$efficient_portfolio,)
```

```
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

```
row.names(mw.portfolio$efficient_portfolio)
```

```
## [1] "DAX" "RWE" "gold" "oil" "$/\200"
```

```
graph.asset.allocation(portfolio=mw.portfolio$efficient_portfolio,
                        sd=mw.portfolio$standarddev, mu=mu, png.name = 'asset-allocation-fig-3-4-C-unbia
```

```

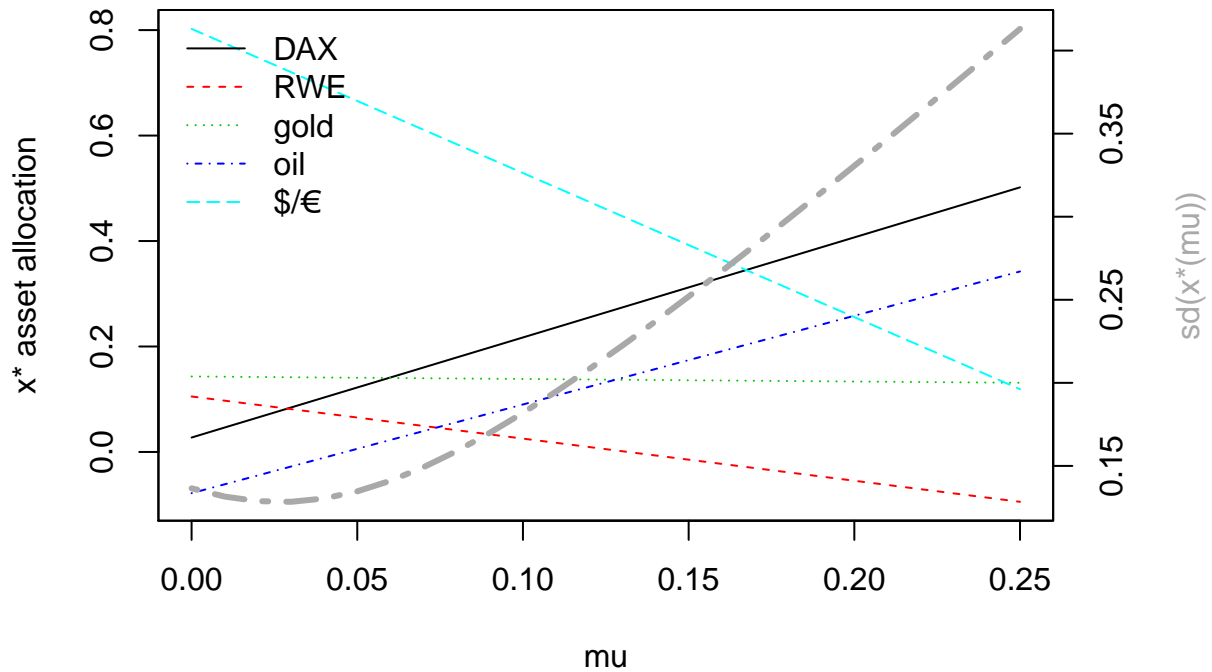
##      [,1]      [,2]      [,3]      [,4]      [,5]
## DAX 0.02739916 0.04637582 0.06535247 0.08432912 0.10330578
## RWE 0.10525296 0.09726392 0.08927488 0.08128584 0.07329680
## gold 0.14319142 0.14271236 0.14223330 0.14175424 0.14127518
## oil -0.07809465 -0.06127758 -0.04446051 -0.02764344 -0.01082637
## $/\200 0.80225111 0.77492548 0.74759986 0.72027424 0.69294861
##      [,6]      [,7]      [,8]      [,9]     [,10]     [,11]
## DAX 0.122282431 0.14125908 0.16023574 0.17921239 0.19818905 0.21716570
## RWE 0.065307762 0.05731872 0.04932968 0.04134064 0.03335160 0.02536256

```

```

## gold 0.140796125 0.14031707 0.13983801 0.13935895 0.13887989 0.13840083
## oil 0.005990696 0.02280776 0.03962483 0.05644190 0.07325897 0.09007604
## $/\200 0.665622987 0.63829736 0.61097174 0.58364611 0.55632049 0.52899487
##      [,12]      [,13]      [,14]      [,15]      [,16]
## DAX 0.23614235 0.255119007 0.274095661 0.293072314 0.31204897
## RWE 0.01737353 0.009384486 0.001395446 -0.006593593 -0.01458263
## gold 0.13792177 0.137442711 0.136963652 0.136484593 0.13600553
## oil 0.10689311 0.123710180 0.140527249 0.157344318 0.17416139
## $/\200 0.50166924 0.474343617 0.447017993 0.419692369 0.39236674
##      [,17]      [,18]      [,19]      [,20]      [,21]
## DAX 0.33102562 0.35000228 0.36897893 0.38795558 0.40693224
## RWE -0.02257167 -0.03056071 -0.03854975 -0.04653879 -0.05452783
## gold 0.13552647 0.13504742 0.13456836 0.13408930 0.13361024
## oil 0.19097846 0.20779553 0.22461259 0.24142966 0.25824673
## $/\200 0.36504112 0.33771550 0.31038987 0.28306425 0.25573862
##      [,22]      [,23]      [,24]      [,25]      [,26]
## DAX 0.42590889 0.44488554 0.46386220 0.48283885 0.50181551
## RWE -0.06251687 -0.07050591 -0.07849495 -0.08648399 -0.09447303
## gold 0.13313118 0.13265212 0.13217306 0.13169400 0.13121494
## oil 0.27506380 0.29188087 0.30869794 0.32551501 0.34233208
## $/\200 0.22841300 0.20108737 0.17376175 0.14643613 0.11911050
## [1] "DAX" "RWE" "gold" "oil" "$/\200"

```



```

## pdf
## 2

```

```
(mw.riskfree = markowitz.portfolio.cash(r,C.unbiased,mu.new,r0))
```

```
## [1] "DAX" "RWE" "gold" "oil" "$/\200"
## $efficient_portfolio
##           [,1]      [,2] [,3]      [,4]      [,5]      [,6]
## DAX -0.03956187 -0.019780937  0  0.019780937  0.03956187  0.059342810
## RWE  0.01378069  0.006890347  0 -0.006890347 -0.01378069 -0.020671041
## gold -0.00254277 -0.001271385  0  0.001271385  0.00254277  0.003814154
## oil  -0.03253980 -0.016269900  0  0.016269900  0.03253980  0.048809701
## $/\200  0.03625005  0.018125025  0 -0.018125025 -0.03625005 -0.054375076
## cash  1.02461370  1.012306850  1  0.987693150  0.97538630  0.963079451
##           [,7]      [,8]      [,9]      [,10]     [,11]
## DAX  0.079123747  0.098904684  0.118685620  0.138466557  0.15824749
## RWE -0.027561388 -0.034451735 -0.041342082 -0.048232429 -0.05512278
## gold  0.005085539  0.006356924  0.007628309  0.008899693  0.01017108
## oil  0.065079601  0.081349502  0.097619402  0.113889302  0.13015920
## $/\200 -0.072500101 -0.090625126 -0.108750151 -0.126875176 -0.14500020
## cash  0.950772601  0.938465752  0.926158902  0.913852052  0.90154520
##           [,12]     [,13]     [,14]     [,15]     [,16]
## DAX  0.17802843  0.19780937  0.21759030  0.23737124  0.25715218
## RWE -0.06201312 -0.06890347 -0.07579382 -0.08268416 -0.08957451
## gold  0.01144246  0.01271385  0.01398523  0.01525662  0.01652800
## oil  0.14642910  0.16269900  0.17896890  0.19523880  0.21150870
## $/\200 -0.16312523 -0.18125025 -0.19937528 -0.21750030 -0.23562533
## cash  0.88923835  0.87693150  0.86462465  0.85231780  0.84001095
##           [,17]     [,18]     [,19]     [,20]     [,21]
## DAX  0.27693311  0.29671405  0.31649499  0.33627592  0.35605686
## RWE -0.09646486 -0.10335520 -0.11024555 -0.11713590 -0.12402625
## gold  0.01779939  0.01907077  0.02034216  0.02161354  0.02288493
## oil  0.22777860  0.24404850  0.26031841  0.27658831  0.29285821
## $/\200 -0.25375035 -0.27187538 -0.29000040 -0.30812543 -0.32625045
## cash  0.82770410  0.81539725  0.80309041  0.79078356  0.77847671
##           [,22]     [,23]     [,24]     [,25]     [,26]
## DAX  0.37583780  0.3956187  0.41539967  0.43518061  0.45496155
## RWE -0.13091659 -0.1378069 -0.14469729 -0.15158763 -0.15847798
## gold  0.02415631  0.0254277  0.02669908  0.02797047  0.02924185
## oil  0.30912811  0.3253980  0.34166791  0.35793781  0.37420771
## $/\200 -0.34437548 -0.3625005 -0.38062553 -0.39875055 -0.41687558
## cash  0.76616986  0.7538630  0.74155616  0.72924931  0.71694246
##           [,27]     [,28]     [,29]     [,30]     [,31]
## DAX  0.47474248  0.49452342  0.5143044  0.53408529  0.55386623
## RWE -0.16536833 -0.17225867 -0.1791490 -0.18603937 -0.19292971
## gold  0.03051323  0.03178462  0.0330560  0.03432739  0.03559877
## oil  0.39047761  0.40674751  0.4230174  0.43928731  0.45555721
## $/\200 -0.43500060 -0.45312563 -0.4712507 -0.48937568 -0.50750071
## cash  0.70463561  0.69232876  0.6800219  0.66771506  0.65540821
##           [,32]     [,33]     [,34]     [,35]     [,36]
## DAX  0.57364717  0.59342810  0.61320904  0.63298998  0.6527709
## RWE -0.19982006 -0.20671041 -0.21360076 -0.22049110 -0.2273814
## gold  0.03687016  0.03814154  0.03941293  0.04068431  0.0419557
## oil  0.47182711  0.48809701  0.50436691  0.52063681  0.5369067
## $/\200 -0.52562573 -0.54375076 -0.56187578 -0.58000081 -0.5981258
## cash  0.64310136  0.63079451  0.61848766  0.60618081  0.5938740
```

##		[,37]	[,38]	[,39]	[,40]	[,41]
## DAX	0.67255185	0.69233279	0.71211372	0.73189466	0.75167560	
## RWE	-0.23427180	-0.24116214	-0.24805249	-0.25494284	-0.26183318	
## gold	0.04322708	0.04449847	0.04576985	0.04704124	0.04831262	
## oil	0.55317661	0.56944651	0.58571641	0.60198631	0.61825621	
## \$/\200	-0.61625086	-0.63437588	-0.65250091	-0.67062593	-0.68875096	
## cash	0.58156711	0.56926026	0.55695341	0.54464656	0.53233971	
##		[,42]	[,43]	[,44]	[,45]	[,46]
## DAX	0.77145653	0.79123747	0.81101841	0.83079934	0.85058028	
## RWE	-0.26872353	-0.27561388	-0.28250423	-0.28939457	-0.29628492	
## gold	0.04958401	0.05085539	0.05212678	0.05339816	0.05466955	
## oil	0.63452611	0.65079601	0.66706591	0.68333581	0.69960571	
## \$/\200	-0.70687598	-0.72500101	-0.74312603	-0.76125106	-0.77937608	
## cash	0.52003286	0.50772601	0.49541916	0.48311231	0.47080546	
##		[,47]	[,48]	[,49]	[,50]	[,51]
## DAX	0.87036122	0.89014215	0.9099231	0.92970403	0.94948496	
## RWE	-0.30317527	-0.31006561	-0.3169560	-0.32384631	-0.33073665	
## gold	0.05594093	0.05721231	0.0584837	0.05975508	0.06102647	
## oil	0.71587561	0.73214551	0.7484154	0.76468531	0.78095522	
## \$/\200	-0.79750111	-0.81562613	-0.8337512	-0.85187618	-0.87000121	
## cash	0.45849861	0.44619176	0.4338849	0.42157807	0.40927122	
##		[,52]	[,53]	[,54]	[,55]	[,56]
## DAX	0.96926590	0.98904684	1.00882777	1.02860871	1.04838965	
## RWE	-0.33762700	-0.34451735	-0.35140769	-0.35829804	-0.36518839	
## gold	0.06229785	0.06356924	0.06484062	0.06611201	0.06738339	
## oil	0.79722512	0.81349502	0.82976492	0.84603482	0.86230472	
## \$/\200	-0.88812623	-0.90625126	-0.92437629	-0.94250131	-0.96062634	
## cash	0.39696437	0.38465752	0.37235067	0.36004382	0.34773697	
##		[,57]	[,58]	[,59]	[,60]	[,61]
## DAX	1.06817058	1.08795152	1.10773246	1.12751339	1.14729433	
## RWE	-0.37207874	-0.37896908	-0.38585943	-0.39274978	-0.39964012	
## gold	0.06865478	0.06992616	0.07119755	0.07246893	0.07374032	
## oil	0.87857462	0.89484452	0.91111442	0.92738432	0.94365422	
## \$/\200	-0.97875136	-0.99687639	-1.01500141	-1.03312644	-1.05125146	
## cash	0.33543012	0.32312327	0.31081642	0.29850957	0.28620272	
##		[,62]	[,63]	[,64]	[,65]	[,66]
## DAX	1.1670753	1.18685620	1.20663714	1.22641808	1.24619901	
## RWE	-0.4065305	-0.41342082	-0.42031116	-0.42720151	-0.43409186	
## gold	0.0750117	0.07628309	0.07755447	0.07882586	0.08009724	
## oil	0.9599241	0.97619402	0.99246392	1.00873382	1.02500372	
## \$/\200	-1.0693765	-1.08750151	-1.10562654	-1.12375156	-1.14187659	
## cash	0.2738959	0.26158902	0.24928217	0.23697532	0.22466847	
##		[,67]	[,68]	[,69]	[,70]	[,71]
## DAX	1.26597995	1.28576089	1.3055418	1.32532276	1.34510370	
## RWE	-0.44098220	-0.44787255	-0.4547629	-0.46165325	-0.46854359	
## gold	0.08136863	0.08264001	0.0839114	0.08518278	0.08645416	
## oil	1.04127362	1.05754352	1.0738134	1.09008332	1.10635322	
## \$/\200	-1.16000161	-1.17812664	-1.1962517	-1.21437669	-1.23250171	
## cash	0.21236162	0.20005477	0.1877479	0.17544107	0.16313422	
##		[,72]	[,73]	[,74]	[,75]	[,76]
## DAX	1.36488464	1.38466557	1.40444651	1.4242274	1.44400838	
## RWE	-0.47543394	-0.48232429	-0.48921463	-0.4961050	-0.50299533	
## gold	0.08772555	0.08899693	0.09026832	0.0915397	0.09281109	
## oil	1.12262312	1.13889302	1.15516292	1.1714328	1.18770272	

```

## $/\200 -1.25062674 -1.26875176 -1.28687679 -1.3050018 -1.32312684
## cash 0.15082737 0.13852052 0.12621367 0.1139068 0.10159997
##      [,77]      [,78]      [,79]      [,80]      [,81]
## DAX  1.46378932 1.48357026 1.50335119 1.52313213 1.54291307
## RWE -0.50988567 -0.51677602 -0.52366637 -0.53055672 -0.53744706
## gold 0.09408247 0.09535386 0.09662524 0.09789663 0.09916801
## oil  1.20397262 1.22024252 1.23651242 1.25278232 1.26905222
## $/\200 -1.34125186 -1.35937689 -1.37750192 -1.39562694 -1.41375197
## cash 0.08929312 0.07698627 0.06467942 0.05237257 0.04006573
##      [,82]      [,83]      [,84]      [,85]
## DAX  1.56269400 1.58247494 1.602255876 1.622036813
## RWE -0.54433741 -0.55122776 -0.558118103 -0.565008450
## gold 0.10043940 0.10171078 0.102982167 0.104253552
## oil  1.28532212 1.30159203 1.317861926 1.334131826
## $/\200 -1.43187699 -1.45000202 -1.468127041 -1.486252067
## cash 0.02775888 0.01545203 0.003145176 -0.009161674
##
## $standarddev
## [1] 0.03500844 0.01750422 0.00000000 0.01750422 0.03500844 0.05251266
## [7] 0.07001688 0.08752110 0.10502532 0.12252954 0.14003376 0.15753798
## [13] 0.17504220 0.19254642 0.21005064 0.22755486 0.24505908 0.26256330
## [19] 0.28006752 0.29757174 0.31507596 0.33258018 0.35008440 0.36758862
## [25] 0.38509283 0.40259705 0.42010127 0.43760549 0.45510971 0.47261393
## [31] 0.49011815 0.50762237 0.52512659 0.54263081 0.56013503 0.57763925
## [37] 0.59514347 0.61264769 0.63015191 0.64765613 0.66516035 0.68266457
## [43] 0.70016879 0.71767301 0.73517723 0.75268145 0.77018567 0.78768989
## [49] 0.80519411 0.82269833 0.84020255 0.85770677 0.87521099 0.89271521
## [55] 0.91021943 0.92772365 0.94522787 0.96273209 0.98023631 0.99774053
## [61] 1.01524475 1.03274897 1.05025319 1.06775741 1.08526163 1.10276585
## [67] 1.12027006 1.13777428 1.15527850 1.17278272 1.19028694 1.20779116
## [73] 1.22529538 1.24279960 1.26030382 1.27780804 1.29531226 1.31281648
## [79] 1.33032070 1.34782492 1.36532914 1.38283336 1.40033758 1.41784180
## [85] 1.43534602

```

```
colSums(mw.riskfree$efficient_portfolio)
```

```

## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [36] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [71] 1 1 1 1 1 1 1 1 1 1 1 1 1 1

```

```
graph.asset.allocation(portfolio=mw.riskfree$efficient_portfolio,
sd=mw.riskfree$standarddev, mu=mu.new, png.name = 'asset-allocation-with-cash-C-')
```

```

##      [,1]      [,2] [,3]      [,4]      [,5]      [,6]
## DAX -0.03956187 -0.019780937 0 0.019780937 0.03956187 0.059342810
## RWE  0.01378069 0.006890347 0 -0.006890347 -0.01378069 -0.020671041
## gold -0.00254277 -0.001271385 0 0.001271385 0.00254277 0.003814154
## oil  -0.03253980 -0.016269900 0 0.016269900 0.03253980 0.048809701
## $/\200 0.03625005 0.018125025 0 -0.018125025 -0.03625005 -0.054375076
## cash 1.02461370 1.012306850 1 0.987693150 0.97538630 0.963079451
##      [,7]      [,8]      [,9]      [,10]      [,11]
## DAX  0.079123747 0.098904684 0.118685620 0.138466557 0.15824749
## RWE -0.027561388 -0.034451735 -0.041342082 -0.048232429 -0.05512278
## gold 0.005085539 0.006356924 0.007628309 0.008899693 0.01017108
## oil  0.065079601 0.081349502 0.097619402 0.113889302 0.13015920

```

```

## $\200 -0.072500101 -0.090625126 -0.108750151 -0.126875176 -0.14500020
## cash 0.950772601 0.938465752 0.926158902 0.913852052 0.90154520
##      [,12]      [,13]      [,14]      [,15]      [,16]
## DAX 0.17802843 0.19780937 0.21759030 0.23737124 0.25715218
## RWE -0.06201312 -0.06890347 -0.07579382 -0.08268416 -0.08957451
## gold 0.01144246 0.01271385 0.01398523 0.01525662 0.01652800
## oil 0.14642910 0.16269900 0.17896890 0.19523880 0.21150870
## $\200 -0.16312523 -0.18125025 -0.19937528 -0.21750030 -0.23562533
## cash 0.88923835 0.87693150 0.86462465 0.85231780 0.84001095
##      [,17]      [,18]      [,19]      [,20]      [,21]
## DAX 0.27693311 0.29671405 0.31649499 0.33627592 0.35605686
## RWE -0.09646486 -0.10335520 -0.11024555 -0.11713590 -0.12402625
## gold 0.01779939 0.01907077 0.02034216 0.02161354 0.02288493
## oil 0.22777860 0.24404850 0.26031841 0.27658831 0.29285821
## $\200 -0.25375035 -0.27187538 -0.29000040 -0.30812543 -0.32625045
## cash 0.82770410 0.81539725 0.80309041 0.79078356 0.77847671
##      [,22]      [,23]      [,24]      [,25]      [,26]
## DAX 0.37583780 0.3956187 0.41539967 0.43518061 0.45496155
## RWE -0.13091659 -0.1378069 -0.14469729 -0.15158763 -0.15847798
## gold 0.02415631 0.0254277 0.02669908 0.02797047 0.02924185
## oil 0.30912811 0.3253980 0.34166791 0.35793781 0.37420771
## $\200 -0.34437548 -0.3625005 -0.38062553 -0.39875055 -0.41687558
## cash 0.76616986 0.7538630 0.74155616 0.72924931 0.71694246
##      [,27]      [,28]      [,29]      [,30]      [,31]
## DAX 0.47474248 0.49452342 0.5143044 0.53408529 0.55386623
## RWE -0.16536833 -0.17225867 -0.1791490 -0.18603937 -0.19292971
## gold 0.03051323 0.03178462 0.0330560 0.03432739 0.03559877
## oil 0.39047761 0.40674751 0.4230174 0.43928731 0.45555721
## $\200 -0.43500060 -0.45312563 -0.4712507 -0.48937568 -0.50750071
## cash 0.70463561 0.69232876 0.6800219 0.66771506 0.65540821
##      [,32]      [,33]      [,34]      [,35]      [,36]
## DAX 0.57364717 0.59342810 0.61320904 0.63298998 0.6527709
## RWE -0.19982006 -0.20671041 -0.21360076 -0.22049110 -0.2273814
## gold 0.03687016 0.03814154 0.03941293 0.04068431 0.0419557
## oil 0.47182711 0.48809701 0.50436691 0.52063681 0.5369067
## $\200 -0.52562573 -0.54375076 -0.56187578 -0.58000081 -0.5981258
## cash 0.64310136 0.63079451 0.61848766 0.60618081 0.5938740
##      [,37]      [,38]      [,39]      [,40]      [,41]
## DAX 0.67255185 0.69233279 0.71211372 0.73189466 0.75167560
## RWE -0.23427180 -0.24116214 -0.24805249 -0.25494284 -0.26183318
## gold 0.04322708 0.04449847 0.04576985 0.04704124 0.04831262
## oil 0.55317661 0.56944651 0.58571641 0.60198631 0.61825621
## $\200 -0.61625086 -0.63437588 -0.65250091 -0.67062593 -0.68875096
## cash 0.58156711 0.56926026 0.55695341 0.54464656 0.53233971
##      [,42]      [,43]      [,44]      [,45]      [,46]
## DAX 0.77145653 0.79123747 0.81101841 0.83079934 0.85058028
## RWE -0.26872353 -0.27561388 -0.28250423 -0.28939457 -0.29628492
## gold 0.04958401 0.05085539 0.05212678 0.05339816 0.05466955
## oil 0.63452611 0.65079601 0.66706591 0.68333581 0.69960571
## $\200 -0.70687598 -0.72500101 -0.74312603 -0.76125106 -0.77937608
## cash 0.52003286 0.50772601 0.49541916 0.48311231 0.47080546
##      [,47]      [,48]      [,49]      [,50]      [,51]
## DAX 0.87036122 0.89014215 0.9099231 0.92970403 0.94948496
## RWE -0.30317527 -0.31006561 -0.3169560 -0.32384631 -0.33073665

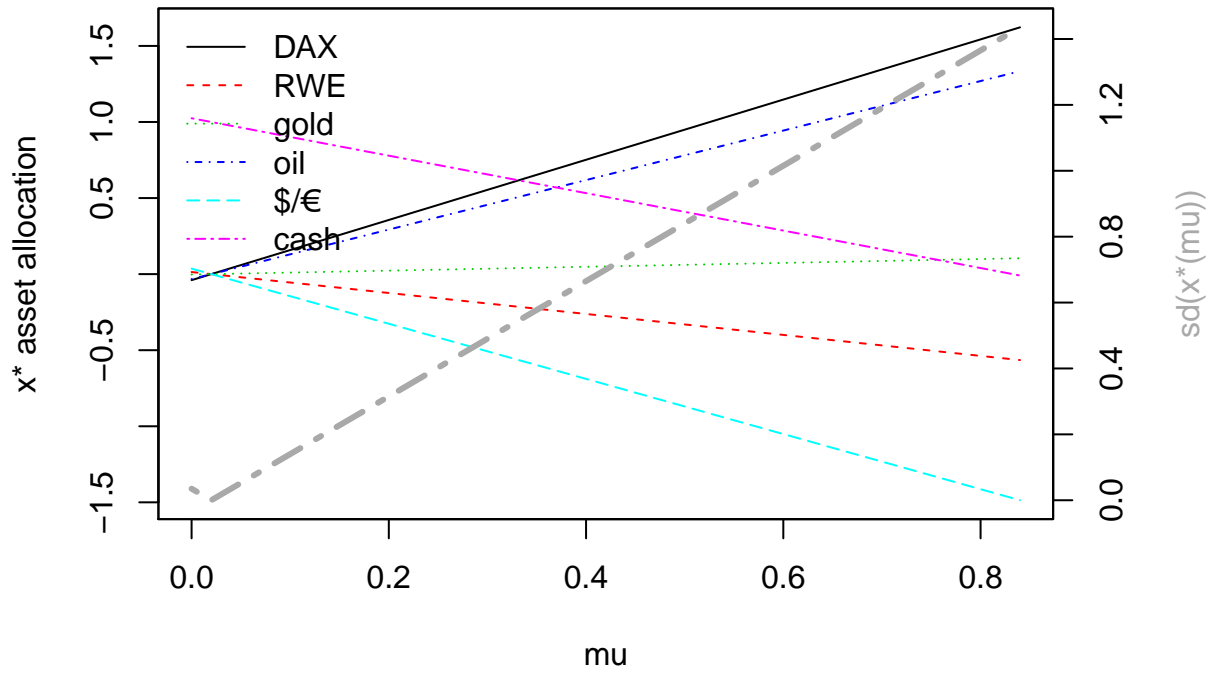
```



```

## gold 0.05594093 0.05721231 0.0584837 0.05975508 0.06102647
## oil 0.71587561 0.73214551 0.7484154 0.76468531 0.78095522
## $\200 -0.79750111 -0.81562613 -0.8337512 -0.85187618 -0.87000121
## cash 0.45849861 0.44619176 0.4338849 0.42157807 0.40927122
##      [,52]      [,53]      [,54]      [,55]      [,56]
## DAX 0.96926590 0.98904684 1.00882777 1.02860871 1.04838965
## RWE -0.33762700 -0.34451735 -0.35140769 -0.35829804 -0.36518839
## gold 0.06229785 0.06356924 0.06484062 0.06611201 0.06738339
## oil 0.79722512 0.81349502 0.82976492 0.84603482 0.86230472
## $\200 -0.88812623 -0.90625126 -0.92437629 -0.94250131 -0.96062634
## cash 0.39696437 0.38465752 0.37235067 0.36004382 0.34773697
##      [,57]      [,58]      [,59]      [,60]      [,61]
## DAX 1.06817058 1.08795152 1.10773246 1.12751339 1.14729433
## RWE -0.37207874 -0.37896908 -0.38585943 -0.39274978 -0.39964012
## gold 0.06865478 0.06992616 0.07119755 0.07246893 0.07374032
## oil 0.87857462 0.89484452 0.91111442 0.92738432 0.94365422
## $\200 -0.97875136 -0.99687639 -1.01500141 -1.03312644 -1.05125146
## cash 0.33543012 0.32312327 0.31081642 0.29850957 0.28620272
##      [,62]      [,63]      [,64]      [,65]      [,66]
## DAX 1.1670753 1.18685620 1.20663714 1.22641808 1.24619901
## RWE -0.4065305 -0.41342082 -0.42031116 -0.42720151 -0.43409186
## gold 0.0750117 0.07628309 0.07755447 0.07882586 0.08009724
## oil 0.9599241 0.97619402 0.99246392 1.00873382 1.02500372
## $\200 -1.0693765 -1.08750151 -1.10562654 -1.12375156 -1.14187659
## cash 0.2738959 0.26158902 0.24928217 0.23697532 0.22466847
##      [,67]      [,68]      [,69]      [,70]      [,71]
## DAX 1.26597995 1.28576089 1.3055418 1.32532276 1.34510370
## RWE -0.44098220 -0.44787255 -0.4547629 -0.46165325 -0.46854359
## gold 0.08136863 0.08264001 0.0839114 0.08518278 0.08645416
## oil 1.04127362 1.05754352 1.0738134 1.09008332 1.10635322
## $\200 -1.16000161 -1.17812664 -1.1962517 -1.21437669 -1.23250171
## cash 0.21236162 0.20005477 0.1877479 0.17544107 0.16313422
##      [,72]      [,73]      [,74]      [,75]      [,76]
## DAX 1.36488464 1.38466557 1.40444651 1.4242274 1.44400838
## RWE -0.47543394 -0.48232429 -0.48921463 -0.4961050 -0.50299533
## gold 0.08772555 0.08899693 0.09026832 0.0915397 0.09281109
## oil 1.12262312 1.13889302 1.15516292 1.1714328 1.18770272
## $\200 -1.25062674 -1.26875176 -1.28687679 -1.3050018 -1.32312684
## cash 0.15082737 0.13852052 0.12621367 0.1139068 0.10159997
##      [,77]      [,78]      [,79]      [,80]      [,81]
## DAX 1.46378932 1.48357026 1.50335119 1.52313213 1.54291307
## RWE -0.50988567 -0.51677602 -0.52366637 -0.53055672 -0.53744706
## gold 0.09408247 0.09535386 0.09662524 0.09789663 0.09916801
## oil 1.20397262 1.22024252 1.23651242 1.25278232 1.26905222
## $\200 -1.34125186 -1.35937689 -1.37750192 -1.39562694 -1.41375197
## cash 0.08929312 0.07698627 0.06467942 0.05237257 0.04006573
##      [,82]      [,83]      [,84]      [,85]
## DAX 1.56269400 1.58247494 1.602255876 1.622036813
## RWE -0.54433741 -0.55122776 -0.558118103 -0.565008450
## gold 0.10043940 0.10171078 0.102982167 0.104253552
## oil 1.28532212 1.30159203 1.317861926 1.334131826
## $\200 -1.43187699 -1.45000202 -1.468127041 -1.486252067
## cash 0.02775888 0.01545203 0.003145176 -0.009161674
## [1] "DAX" "RWE" "gold" "oil" "$\200" "cash"

```



## pdf  
## 2