Shnol's theorem for general Schrödinger operators

Peter Stollmann

OTAMP, Bedlewo, 21.06.2008

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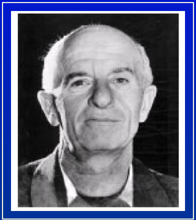
Overview



- The proof
- The results
- The set-up
- Return of the proof
- References and more results
- Thank the organizers

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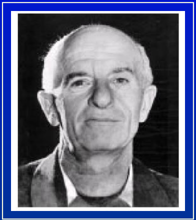

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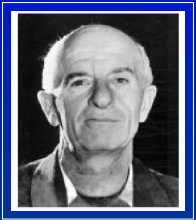

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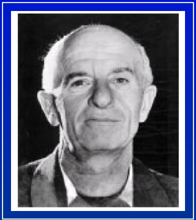

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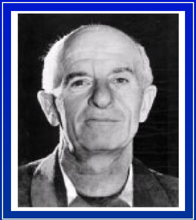

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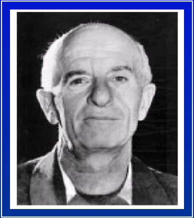

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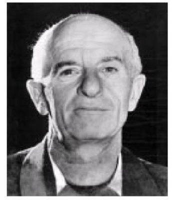

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A remix of results obtained in collaboration with Anne Boutet de Monvel and Daniel Lenz.

The proof

Assume

$$(H + V)u = \lambda u,$$

i.e., u is an eigensolution. For suitable cut-off functions η_n, $n \in \mathbb{N}$:

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so that $\lambda \in \sigma(H+V)$.

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Conclusion

If H is a suitable PDO and u a weak solution of $(H+V)u = \lambda u$ that doesn't grow too rapidly then $\lambda \in \sigma(H+V)$.

Results of this type are known as Shnol's theorem; Shnol 1957, Simon 1981; particularly important for us Kuchment 2005 (for quantum graphs).

Questions

- *What if $D(H)$ is not known? What about Weyl's criterion?*
- *What are good cut-off functions?*
- *How do we measure size/distance?*

We give an answer in the case that H is the generator of a strictly local Dirichlet form. That includes 2nd order PDO with singular coefficients.

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then $\lambda \in \sigma(H + V)$.

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We call u *subexponentially bounded* if, for some $x_0 \in X$ and all $\alpha > 0$:

$$e^{-\alpha \operatorname{dist}(x_0, \cdot)} u \in L^2(X).$$

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Cut-off functions

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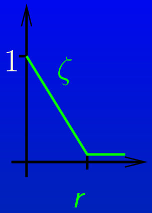

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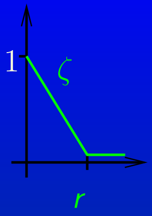

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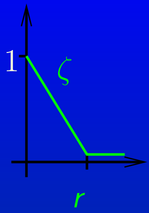

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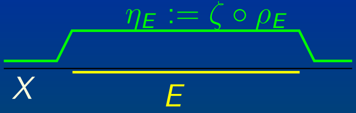
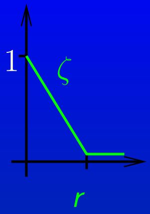


Cut-off functions

composed with



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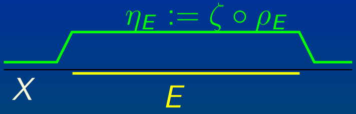

s.t. $\chi_E \leq \eta_E \leq \chi_{B_r(E)}$ and $d\Gamma(\eta_E) \leq r^{-2}\chi_{A(r,E)}dm$.

Weak solutions

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$$= \int_X ud\Gamma(\eta,v) - \int_X vd\Gamma(u,\eta) \text{ by Leibniz' rule.}$$

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$$\sup_{v \in D(t), \|v\|_t \leq 1} \left| (t - \lambda)(\frac{u_n}{\|u_n\|}, v) \right| \longrightarrow 0 \text{ as } n \longrightarrow \infty$$

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&\leq \frac{c}{\|u\cdot\chi_{E_n}\|}\|u\cdot\chi_{A(r,E_n)}\| \longrightarrow 0
\end{aligned}$$

More results

Dirichlet forms: Beurling, Deny (1956)Fukushima (1980)
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- Combes - Thomas estimate for $H + V$ in terms of the intrinsic metric. Eigenfunction expansion for $H + V$ (Boutet de Monvel, S. '03).
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