

Band edge
localization

Peter
Stollmann

What's new?

Fluctuation
boundaries

The result

Applications

Localization near fluctuation boundaries via the fractional moment method

Peter Stollmann

Oberwolfach, Workshop "Transport in Multi-Dimensional
Random Schrödinger Operators", 5.3.-10.3. 2007



Outline

This is about joint work with

A. Boutet de Monvel, S. Naboko and G. Stolz
eprint mparc 05-324, to appear in J. Anal. Math.

- What's new?
- Fluctuation boundaries. This is the key notion.
- The precise results.
- Applications.
- Some details – blackboard

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- We present a fractional moment method that works without a covering condition.
- We can treat quite arbitrary geometries, quite arbitrary background ...
- ... as long as the ground state energy is induced by the random perturbation.
- We can handle certain models that have not been treated by multiscale analysis.

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Fluctuation boundaries

- This notion is classical; it refers to extreme spectral energies that are subtle to changes of the randomness.
- At Fluctuation boundaries one expects Lifshitz tails

In our results we assume $d \leq 3$ and rely upon the following assumptions, which guarantee self-adjointness and lower semi-boundedness of all the Schrödinger operators in question.

- (A1) The background potential $V_0 \in L^2_{\text{loc},\text{unif}}(\mathbb{R}^d)$ is real-valued, $H_0 := -\Delta + V_0$.
- (A2) The set $\mathcal{I} \subset \mathbb{R}^d$, where the random impurities are located, is uniformly discrete, i.e., $\inf\{|\alpha - \beta| : \alpha \neq \beta \in \mathcal{I}\} =: r_{\mathcal{I}} > 0$.

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(A3) The random couplings η_α , $\alpha \in \mathcal{I}$, are independent random variables supported on $[0, \eta_{\max}]$ for some $\eta_{\max} > 0$ and with absolutely continuous distribution of bounded density ρ_α with a uniform bound $\sup_\alpha \|\rho_\alpha\|_\infty =: M_\rho < \infty$. The single site potentials U_α , $\alpha \in \mathcal{I}$ satisfy

$$c_U \chi_{\Lambda_{r_U}}(\alpha) \leq U_\alpha \leq C_U \chi_{\Lambda_{R_U}}(\alpha)$$

for all α with $c_U, C_U, r_U, R_U > 0$ independent of α .

$$V_\omega(x) = \sum_{\alpha \in \mathcal{I}} \eta_\alpha(\omega) U_\alpha(x)$$

and

$$H := H(\omega) := H_0 + V_\omega \text{ in } L^2(\mathbb{R}^d).$$

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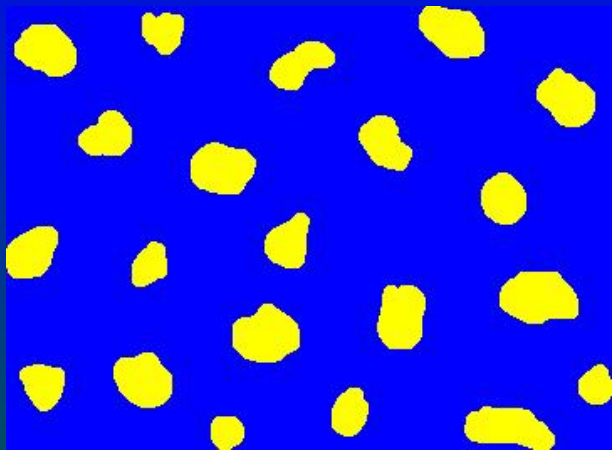
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The random bumps may have different shape and need not be situated at the sites of a lattice.



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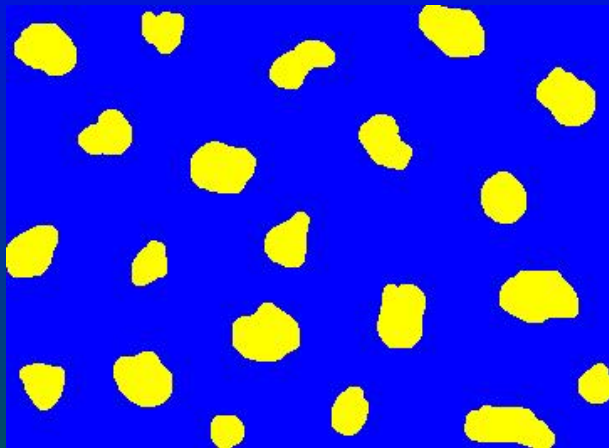
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Fluctuation boundaries

The most important condition expresses the fact that the ground state energy comes from those realizations of the potential that vanish on large sets:

(A4) Denote $E_0 := \inf \sigma(H_0) \leq \inf \sigma(H(\omega))$ and let

$$H_F := H_0 + \eta_{\max} \sum_{\alpha \in I} U_\alpha,$$

the subscript F standing for full coupling.

Assume that E_0 is a *fluctuation boundary* in the sense that

- (i) $E_F := \inf \sigma(H_F) > E_0$, and
- (ii) There is $m \in (0, 2)$ and L^* such that for $m_d := 42 \cdot d$, all $L \geq L^*$ and $x \in \mathbb{Z}^d$

$$\mathbb{P}(\sigma(H^{\wedge L(x)}(\omega) \cap [E_0, E_0 + L^{-m}] \neq \emptyset) \leq L^{-m_d}.$$

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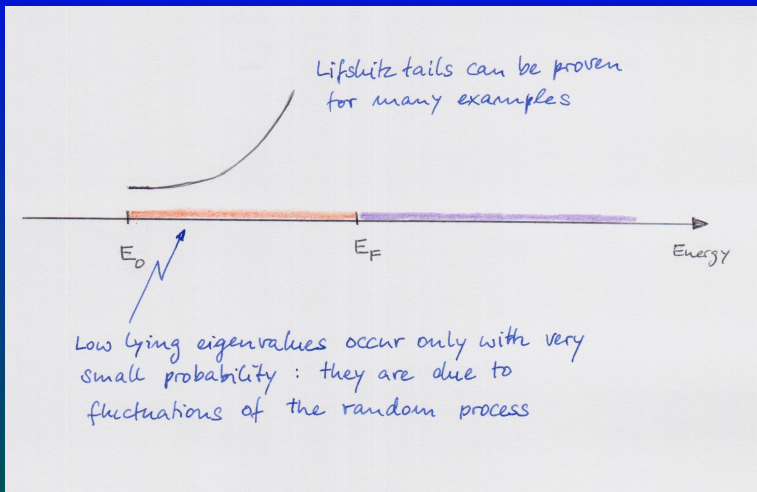
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Our main result is

Theorem

Let $d \leq 3$ and assume (A1)-(A4). Then there exist $\delta > 0$, $0 < s < 1$, $\mu > 0$ and $C < \infty$ such that for $I := [E_0, E_0 + \delta]$, all open sets $G \subset \mathbb{R}^d$ and $x, y \in \mathbb{R}^d$,

$$\sup_{E \in I, \varepsilon > 0} \mathbb{E}(\|\chi_x(H^G - E - i\varepsilon)^{-1}\chi_y\|^s) \leq C e^{-\mu|x-y|}. \quad (1)$$

Exponential decay of fractional moments of the resolvent as described by (1) implies spectral and dynamical localization.

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Exponential decay of fractional moments of the resolvent as described by (1) implies spectral and dynamical localization.

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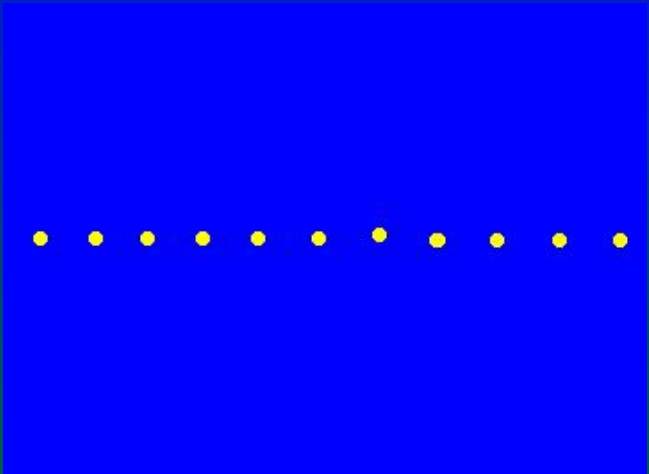
The result

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Surface potentials

The first application is concerned with surface potentials: The necessary estimates at the fluctuation boundary was given by Kirsch and Warzel; we use an extension of their result:



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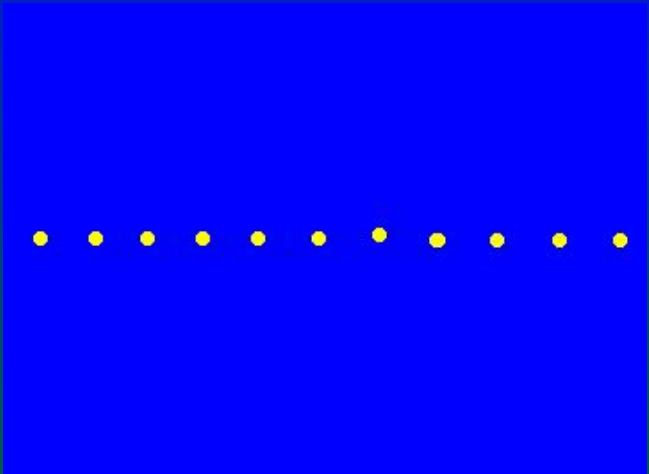
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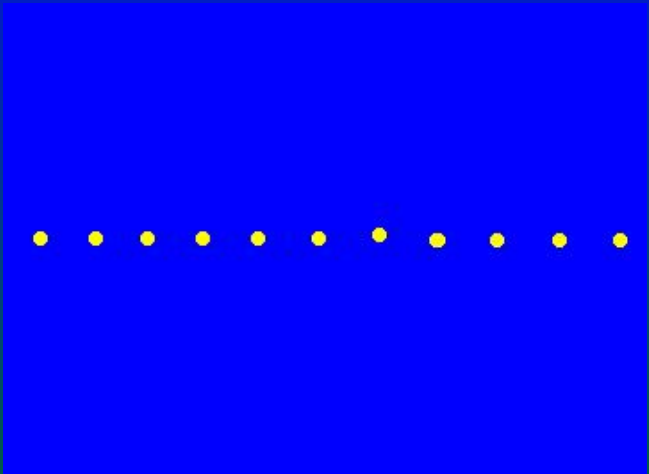
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Surface potentials

What is it that we need to verify the fluctuation boundary assumption in the surface case? As usual, the background is assumed to be partially periodic:

(B1) Fix $1 \leq d_1 \leq d$ and write $\mathbb{R}^d = \mathbb{R}^{d_1} \times \mathbb{R}^{d_2}$, $x = (x_1, x_2)$; assume that $V_0 \in L^2_{\text{loc,unif}}(\mathbb{R}^d)$ is real-valued and periodic with respect to the first variable, i.e.,

$$V_0(x_1 + m, x_2) = V_0(x_1, x_2) \text{ for } m \in \mathbb{Z}^{d_1}.$$

Denote $H_0 := -\Delta + V_0$.

For V_0, H_0 as in (B1) we get a direct integral decomposition

$$H_0 = (2\pi)^{-d_1} \int_{\mathbb{T}^{d_1}}^{\oplus} h_\theta \, d\theta,$$

where $\mathbb{T}^{d_1} = \mathbb{R}^{d_1} / (2\pi\mathbb{Z})^{d_1}$ is the d_1 -dimensional torus and

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Surface potentials

$$h_\theta = -\Delta + V_0 \text{ in } L^2(S_1)$$

with θ -periodic boundary conditions on the unit strip

$S_1 = \Lambda_1(0) \times \mathbb{R}^{d_2}$. We now fix the assumption

(B2)

$$\inf \sigma(h_0) < \inf \sigma_{\text{ess}}(h_0).$$

It is well known that under (B2) we have that

$$E_0 := \inf \sigma(H_0) = \inf \sigma(h_0)$$

and there is a positive eigensolution ψ_0 of the distributional equation

$$H_0\psi_0 = E_0\psi_0,$$

see Kirsch & Warzel.

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$$h_\theta = -\Delta + V_0 \text{ in } L^2(S_1)$$

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Finally, our random perturbation is assumed to satisfy

(B3) The set $\mathcal{I} \subset \mathbb{R}^d$, where the random impurities are located, is uniformly discrete, i.e.,
 $\inf\{|\alpha - \beta| : \alpha \neq \beta \in \mathcal{I}\} =: r_{\mathcal{I}} > 0$. Moreover \mathcal{I} is dense near the surface $\mathbb{R}^{d_1} \times \{0\}$ in the sense that there exist $R_{\perp}, c_{\perp} > 0$ such that for L large enough and $x_1 \in \mathbb{R}^{d_1}$:

$$\#\left[\mathcal{I} \cap \left(\Lambda_L(x_1) \times \Lambda_{R_{\perp}}(0)\right)\right] \geq c_{\perp} L^{d_1}.$$

(B1)-(B3) \implies (A4)

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Anderson plus displacement

The second application is concerned with Anderson plus displacement potentials: They were first studied by Combes and Hislop who needed a covering condition; we use an extension of the standard Lifshitz tails estimate:

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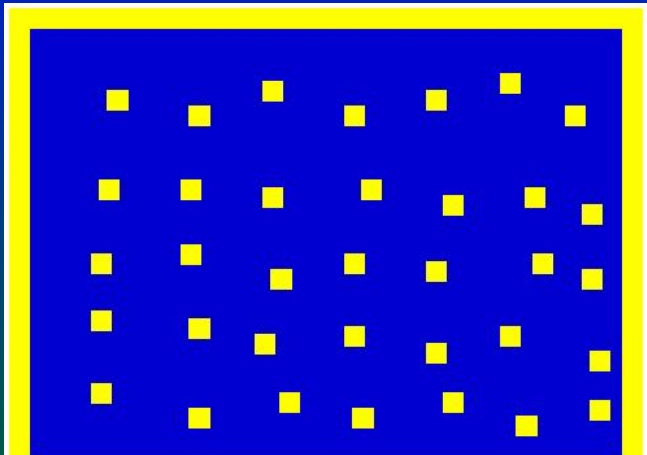
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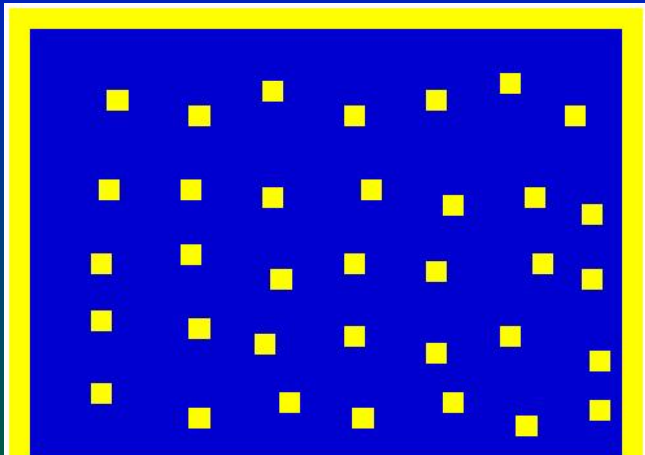
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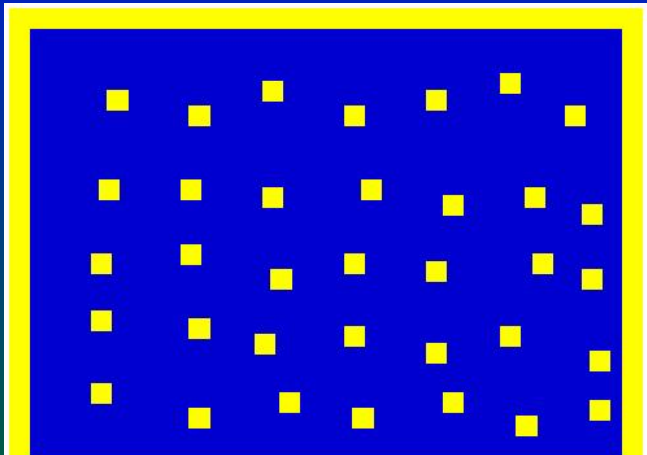
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Anderson plus displacement

By considering the special case $d_1 = d$, the results of the previous Section also cover “usual” Anderson models, sometimes also called alloy models. Note that in this case (B1) means periodic background and (B2) becomes trivial. The obtained bounds hold uniformly in the geometric parameters describing the random potential. This can then be applied to models with random displacements. E.g., we can treat the following model introduced in Combes & Hislop and further studied by Zenk.

(D1) $V_0 \in L^2_{\text{loc,unif}}(\mathbb{R}^d)$ is real-valued and periodic.

(D3) Let $\eta_j, j \in \mathbb{Z}^d$ be independent random couplings, defined on a probability space Ω with distribution ρ_j and U_j as in (A3).

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(D4) Let $x_j, j \in \mathbb{Z}^d$ be independent random vectors of length at most $\frac{1}{3}$ in \mathbb{R}^d ; denote the corresponding probability space by $\tilde{\Omega}$.

Define

$$H(\omega, \tilde{\omega}) := -\Delta + V_0 + \sum_{j \in \mathbb{Z}^d} \eta_j(\omega) U_j(\cdot - j - x_j(\tilde{\omega})).$$

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Corollary

Assume (D1), (D3), (D4). Then, for $H(\omega, \tilde{\omega})$ as above there exist $\delta > 0$, $0 < s < 1$, $\mu > 0$ and $C < \infty$ such that for $I := [E_0, E_0 + \delta]$, all open sets $G \subset \mathbb{R}^d$ and $x, y \in \mathbb{R}^d$

$$\sup_{E \in I, \varepsilon > 0} \tilde{\mathbb{E}} \mathbb{E} (\| \chi_x (H^G - E - i\varepsilon)^{-1} \chi_y \|^s) \leq C e^{-\mu|x-y|}. \quad (2)$$

In particular, the following consequences hold:

- (a) The spectrum of H in I is almost surely pure point with exponentially decaying eigenfunctions.
- (b) There are $\mu > 0$ and $C < \infty$ such that for all $x, y \in \mathbb{Z}^d$,

$$\tilde{\mathbb{E}} \mathbb{E} \left(\sup_{t \in \mathbb{R}} \| \chi_x e^{-itH} P_I(H) \chi_y \| \right) \leq C e^{-\mu|x-y|}. \quad (3)$$

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