Analysis I, 9. Übung

WS 2013/14 http://www.tu-chemnitz.de/~peju/lehre/ana.html

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1. Untersuchen Sie mit Hilfe des Dirichlet- oder Leibniz-Kriteriums auf Konvergenz ($\alpha \in \mathbb{R}$):

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ (b) $\displaystyle\sum_{n=1}^{\infty} b_n \sin(n\alpha)$ ($(b_n)_{n=1}^{\infty}$ monotone Nullfolge) (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n\alpha)}{n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{2} + \ldots + \frac{1}{n}\right) \frac{\sin(n\alpha)}{n}$ (e) **(HA)** $\displaystyle\sum_{n=1}^{\infty} (-1)^n (\sqrt[n]{n} - 1)$

   (f) **(HA)** $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left[e - \left(1 + \frac{1}{n}\right)^n\right]$

2. Untersuchen Sie folgende Reihen auf Konvergenz:

   (a) $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{\frac{1}{n}}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n(n-1)}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{n!}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{2^n}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(\arctan n)^n}{2^n}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$ (g) $\displaystyle 1 + \frac{3}{2} + \frac{1}{2^2} + \frac{3}{2^3} + \cdots + \frac{1}{2^{2n}} + \frac{3}{2^{2n+1}} + \cdots$ (h) $\displaystyle\sum_{n=1}^{\infty} n^2 \sin\frac{2}{n} \tan\frac{5}{n}$

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{2n^3-1}$ (j) $\displaystyle\sum_{n=0}^{\infty} \frac{1+(-1)^n n^2}{2+n^2}$ (k) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ (l) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ (m) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2 5^n}{(2n)!}$

   (n) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ (o) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n!}}$ (p) **(HA)** $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\pi}{4} - \frac{1}{6n}\right)^n$ (q) $\displaystyle\sum_{n=1}^{\infty} (-1)^n (\sqrt[n]{3} - 1)$

   (r) $\displaystyle\sum_{n=1}^{\infty} \sin\frac{1}{n} \cos^2\frac{1}{n}$ (s) **(HA)** $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n}$ (t) $\displaystyle\sum_{n=1}^{\infty} \sin\left[\left(n + \frac{1}{n}\right)\pi\right]$

   (u) **(HA)** $\displaystyle\sum_{n=1}^{\infty} 2^n \sin\frac{1}{3^n}$ (v) $\displaystyle\sum_{n=1}^{\infty} (\ln(n+1) - \ln n)$ (w) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(a+bn)^s}$ ($s \in \mathbb{R}$, $a, b \geq 0$)

   (x) **(HA)** $\displaystyle\sum_{n=1}^{\infty} \sin\frac{1}{\sqrt{n}} \tan\frac{1}{\sqrt{n}}$ (y) **(HA)** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n^2+1)}}$ (z) **(HA)** $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

   (ä) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos nx - \cos(n+1)x}{n}$ ($x \in \mathbb{R}$) (ö) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^\rho}$ ($\rho > 0$) (ü) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$

   (α) **(HA)** $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$ (β) **(HA)** $\displaystyle\left(\sum_{k=1}^{2014} \frac{k!}{5^k}\right) + \sum_{n=1}^{\infty} \frac{n}{5^n}$ (γ) **(HA)** $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}(\sqrt{n+1} - \sqrt{n})$

3. Zeigen Sie, dass das Cauchy-Produkt zweier absolut konvergenter Reihen absolut konvergiert.