Aufgabe 22.25

Lösen Sie das inhomogene Differenzialgleichungssystem $\begin{array}{l} \dot{x} = -2y \\ \dot{y} = -2x+2e^{3t} \end{array}$!

Lösung:

homogen: $\dot{\vec{x}}(t) = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix} \vec{x}(t),\ \begin{vmatrix} -\lambda & -2 \\ -2 & -\lambda \end{vmatrix} = \lambda^2 - 4 = 0,\ \lambda_{1/2} = 2, -2,$

EV zu $\lambda_1 = 2$: $\begin{array}{rr} -2 & -2 \\ -2 & -2 \\ \hline 1 & 1 \\ 0 & 0 \end{array}$ EV $C\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

EV zu $\lambda_2 = -2$: $\begin{array}{rr} 2 & -2 \\ -2 & 2 \\ \hline 1 & -1 \\ 0 & 0 \end{array}$ EV $D\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\vec{x}_{\text{hom}}(t) = C\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t} + D\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$$

inhomogen: rechte Seite $\begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{3t}$, $\lambda = 3 \neq \pm 2$, keine Resonanz,

Ansatz: $\vec{x}(t) = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{3t},\ \dot{\vec{x}}(t) = 3\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{3t}$

$$3\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{3t} = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{3t} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{3t}, \quad \begin{array}{l} 3a_1 = -2a_2 \\ 3a_2 = -2a_1 + 2 \end{array} \quad \begin{array}{l} 3a_1 + 2a_2 = 0 \\ 2a_1 + 3a_2 = 2 \end{array}$$

$$\begin{array}{l} 6a_1 + 4a_2 = 0 \\ 6a_1 + 9a_2 = 6 \end{array} \quad 5a_2 = 6,\ a_2 = \frac{6}{5},\ a_1 = -\frac{2}{3}a_2 = -\frac{4}{5},$$

$$\underline{\underline{\vec{x}(t) = \frac{2}{5}\begin{pmatrix} -2 \\ 3 \end{pmatrix} e^{3t} + C\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t} + D\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}}}$$