Aufgabe 22.22

Lösen Sie die Anfangswertaufgabe $\vec{y}'(x) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{pmatrix} \vec{y}(x)$, $\vec{y}(0) = \begin{pmatrix} 4 \\ 7 \\ 15 \end{pmatrix}$!

Lösung:

$$|A - \lambda E| = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 6 & -11 & 6-\lambda \end{vmatrix} = \lambda^2(6-\lambda) + 6 - 11\lambda = -\lambda^3 + 6\lambda^2 - 11\lambda + 6 = 0$$

$$\lambda_1 = 1, \quad (\lambda^3 - 6\lambda^2 + 11\lambda - 6) : (\lambda - 1) = \lambda^2 - 5\lambda + 6 \ , \ \lambda_{2/3} = \frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{24}{4}} = 2; 3$$

$$\begin{array}{l} \underline{\lambda^3 - \ \lambda^2} \\ \quad -5\lambda^2 + 11\lambda - 6 \\ \quad \underline{-5\lambda^2 + \ 5\lambda} \\ \qquad\qquad 6\lambda - 6 \end{array}$$

Eigenvektoren

zu $\lambda = 1$:			zu $\lambda = 2$:			zu $\lambda = 3$:		
-1	1	0	-2	1	0	-3	1	0
0	-1	1	0	-2	1	0	-3	1
6	-11	5	6	-11	4	6	-11	3
-1	1	0	-2	1	0	-3	1	0
0	-1	1	0	-2	1	0	-3	1
0	-5	5	0	-8	4	0	-9	3
-1	1	0	-2	1	0	-3	1	0
0	-1	1	0	-2	1	0	-3	1
0	0	0	0	0	0	0	0	0
-1	1	0	-2	1	0	-3	1	0
-1	0	1	-4	0	1	-9	0	1

$$\text{EV } A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad\qquad \text{EV } B \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \qquad\qquad \text{EV } C \begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix}$$

Lösung des Differenzialgleichungssystems:

$$\vec{y}(x) = A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^x + B \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} e^{2x} + C \begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix} e^{3x}$$

$$\vec{y}(0) = A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + B \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + C \begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 15 \end{pmatrix}$$

1	1	1	4		1	1	1	4		1	1	0	3
1	2	3	7		0	1	2	3		0	1	0	1
1	4	9	15		0	0	2	2		0	0	1	1
1	1	1	4		1	1	1	4		1	0	0	2
0	1	2	3		0	1	2	3		0	1	0	1
0	3	8	11		0	0	0	1		0	0	1	1

$$A = 2, B = 1, C = 1, \text{ d.h. } \vec{y}(x) = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} e^x + \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} e^{2x} + \begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix} e^{3x}$$