Aufgabe 22.10

Ermitteln Sie die Lösung des Differenzialgleichungssystems $\begin{array}{l}\dot{x} = \ \ 4x-5y \\ \dot{y} = 10x+6y\end{array}$, für die $x(0)=10$ und $\dot{x}(0)=50$ gilt!

Lösung:

$$\begin{vmatrix} 4-\lambda & -5 \\ 10 & 6-\lambda \end{vmatrix} = (4-\lambda)(6-\lambda)+50 = 24-10\lambda+\lambda^2+50 = \lambda^2-10\lambda+74 = 0,$$

$$\lambda_{1/2} = 5 \pm \sqrt{25-74} = 5 \pm \sqrt{-49} = 5 \pm 7\mathrm{i}$$

$$x(t) = A\mathrm{e}^{5t}\cos 7t + B\mathrm{e}^{5t}\sin 7t,$$
$$\dot{x}(t) = 5A\mathrm{e}^{5t}\cos 7t - 7A\mathrm{e}^{5t}\sin 7t + 5B\mathrm{e}^{5t}\sin 7t + 7B\mathrm{e}^{5t}\cos 7t,$$
$$x(0) = A = 10,\ \ \dot{x}(0) = 5A + 7B = 50 + 7B = 50 \ \Rightarrow\ B = 0$$

$$x(t) = 10\mathrm{e}^{5t}\cos 7t,\ \ \dot{x}(t) = 50\mathrm{e}^{5t}\cos 7t - 70\mathrm{e}^{5t}\sin 7t$$

$$\dot{x} = 4x - 5y \ \Rightarrow\ y = \frac{1}{5}(4x - \dot{x}) = \frac{1}{5}(40\mathrm{e}^{5t}\cos 7t - 50\mathrm{e}^{5t}\cos 7t + 70\mathrm{e}^{5t}\sin 7t),$$
$$y(t) = -2\mathrm{e}^{5t}\cos 7t + 14\mathrm{e}^{5t}\sin 7t$$

$$\underline{\underline{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ -2 \end{pmatrix}\mathrm{e}^{5t}\cos 7t + \begin{pmatrix} 0 \\ 14 \end{pmatrix}\mathrm{e}^{5t}\sin 7t}}$$