Aufgabe 21.39

Lösen Sie die Anfangswertaufgaben

a) $y''' - 3y'' + 4y = 0,\ \ y(0) = 0,\ \ y'(0) = 1,\ \ y''(0) = 4,$
b) $\ddot{s} + 2\dot{s} + 2s = 0,\ \ s(0) = \dot{s}(0) = 1$!

Lösung:

a) $\lambda^3 - 3\lambda^2 + 4 = 0,\ \lambda_1 = -1,\ \ (\lambda^3 - 3\lambda^2 + 4) : (\lambda + 1) = \lambda^2 - 4\lambda + 4 = (\lambda - 2)^2,\ \lambda_{2/3} = 2$

$$\begin{array}{l} \lambda^3 + \lambda^2 \\ \hline -4\lambda^2 + 4 \\ -4\lambda^2 - 4\lambda \\ \hline 4\lambda + 4 \\ 4\lambda + 4 \\ \hline 0 \end{array}$$

$$\begin{array}{rcl} y(x) &=& Ae^{-x} + (B + Cx)e^{2x} \\ y'(x) &=& -Ae^{-x} + (C + 2B + 2Cx)e^{2x} \\ y''(x) &=& Ae^{-x} + (2C + 2C + 4B + 4Cx)e^{2x} \end{array} \qquad \begin{array}{rcll} y(0) &=& A + B = 0 & A = -B, \\ y'(0) &=& -A + 2B + C = 1 & 3B + C = 1, \\ y''(0) &=& A + 4B + 4C = 4 & \end{array}$$

$$C = 1 - 3B,\ -B + 4B + 4 - 12B = 4,\ B = 0,\ A = 0,\ C = 1$$

Lösung der AWA: $y(x) = xe^{2x}$

b) $\lambda^2 + 2\lambda + 2 = 0,\ \ \lambda_{1/2} = -1 \pm \sqrt{1-2} = -1 \pm \mathrm{i}$

$$\begin{array}{ll} s(t) = e^{-t}(A\cos t + B\sin t) & s(0) = A = 1 \\ \dot{s}(t) = e^{-t}(-A\cos t - B\sin t - A\sin t + B\cos t) & \dot{s}(0) = -A + B = 1,\quad B = 2 \end{array}$$

Lösung der AWA: $s(t) = e^{-t}(\cos t + 2\sin t)$