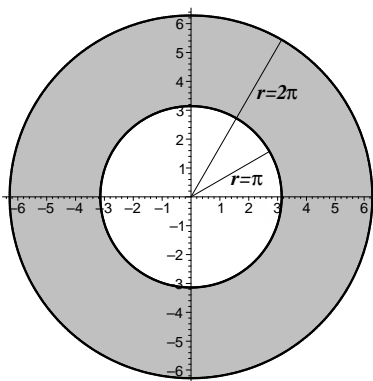


Aufgabe 20.16

Berechnen Sie $\iint_B \sin \sqrt{x^2 + y^2} \, dx \, dy$, wobei B durch die Kreise $x^2 + y^2 = \pi^2$ und $x^2 + y^2 = 4\pi^2$ begrenzt sei!

Lösung:



$$x = r \cos \varphi, \quad y = r \sin \varphi, \quad x^2 + y^2 = r^2,$$
$$B = \{(r, \varphi) : \pi \leq r \leq 2\pi, 0 \leq \varphi < 2\pi\},$$

$dx \, dy = r \, dr \, d\varphi$ (wie bei Aufgabe 20.14)

$$\begin{aligned} \iint_B \sin \sqrt{x^2 + y^2} \, dx \, dy &= \iint_B \sin r \, r \, dr \, d\varphi = \int_0^{2\pi} \left(\int_{\pi}^{2\pi} r \sin r \, dr \right) d\varphi \\ &= \int_0^{2\pi} \left[-r \cos r + \int \cos r \, dr \right]_{\pi}^{2\pi} d\varphi = \int_0^{2\pi} [-r \cos r + \sin r]_{\pi}^{2\pi} d\varphi \\ &= \int_0^{2\pi} (-2\pi - (-\pi(-1))) d\varphi = -3\pi \varphi \Big|_0^{2\pi} = \underline{\underline{-6\pi^2}} \end{aligned}$$