Aufgabe 20.13

Berechnen Sie $\int_0^1 \left(\int_0^2 \left(\int_0^3 \sin(x+y+z)\,\mathrm{d}z \right) \mathrm{d}y \right) \mathrm{d}x$!

Lösung:

$$\int_0^1 \left(\int_0^2 \left(\int_0^3 \sin(x+y+z)\,\mathrm{d}z \right) \mathrm{d}y \right) \mathrm{d}x = \int_0^1 \int_0^2 \Big[-\cos(x+y+z) \Big]_{z=0}^{z=3} \mathrm{d}y\,\mathrm{d}x$$

$$= \int_0^1 \int_0^2 (\cos(x+y) - \cos(x++y+3))\,\mathrm{d}y\,\mathrm{d}x = \int_0^1 \Big[\sin(x+y) - \sin(x+y+3) \Big]_{y=0}^{y=2} \mathrm{d}x$$

$$= \int_0^1 \big(\sin(x+3) - \sin(x+5) - \sin x + \sin(x+3) \big)\,\mathrm{d}x$$

$$= \Big[-\cos(x+2) + \cos(x+5) + \cos x - \cos(x+3) \Big]_{x=0}^{x=1}$$

$$= -\cos 3 + \cos 6 + \cos 1 - \cos 4 + \cos 2 - \cos 5 - \cos 0 + \cos 3$$

$$= \cos 6 - \cos 5 - \cos 4 + \cos 2 + \cos 1 - \cos 0 \approx 0{,}4543$$