Aufgabe 18.21

Sei $u=f(\xi,\eta)$ mit $\xi=x+y,\ \eta=x-y$. Berechnen Sie $\dfrac{\partial u}{\partial x}$ und $\dfrac{\partial u}{\partial y}$!

Lösung:

Kettenregel: Für $H(t_1,t_2,\ldots,t_n)=F\left(x_1(t_1,t_2,\ldots,t_n),x_2(t_1,t_2,\ldots,t_n),\ldots,x_m(t_1,t_2,\ldots,t_n)\right)$ gilt

$$\frac{\partial H}{\partial t_i}=\sum_{j=1}^{m}\frac{\partial F}{\partial x_j}\frac{\partial x_j}{\partial t_i},\quad i=1,2,\ldots,n.$$

$$\frac{\partial u}{\partial x}=\frac{\partial f}{\partial \xi}\frac{\partial \xi}{\partial x}+\frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial x}=\frac{\partial f}{\partial \xi}+\frac{\partial f}{\partial \eta}$$

$$\frac{\partial u}{\partial y}=\frac{\partial f}{\partial \xi}\frac{\partial \xi}{\partial y}+\frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial y}=\frac{\partial f}{\partial \xi}-\frac{\partial f}{\partial \eta}$$