Aufgabe 18.19

Sei $F(x)=f\big(x,g(x)\big)$. Berechnen Sie $F'(x)$

a) allgemein,

b) für $f(x,y)=\ln(x+y),\ g(x)=\sin x,\ x=\dfrac{\pi}{2}$!

Lösung:

a) $F'(x)=\dfrac{\partial f}{\partial x}\dfrac{\mathrm{d}x}{\mathrm{d}x}+\dfrac{\partial f}{\partial g}\dfrac{\mathrm{d}g}{\mathrm{d}x}=\dfrac{\partial f}{\partial x}+\dfrac{\partial f}{\partial g}g'(x)$

b) $F(x)=f\big(x,g(x)\big)=\ln\big(x+g(x)\big)=\ln(x+\cos x)$

$F'(x)=\dfrac{1}{x+g(x)}+\dfrac{1}{x+g(x)}\cos x=\dfrac{1+\cos x}{x+\sin x},\quad F'\left(\dfrac{\pi}{2}\right)=\dfrac{1}{\frac{\pi}{2}+1}=\dfrac{2}{\pi+2}$