Aufgabe 16.15

Gegeben sei die Matrix $\boldsymbol{A} = \begin{pmatrix} 3 & -10 & -10 \\ 0 & 3 & 0 \\ 0 & -5 & -2 \end{pmatrix}$.

a) Berechnen Sie die Determinante!
b) Invertieren Sie die Matrix!
c) Bestimmen Sie die Eigenwerte und Eigenvektoren!

Lösung:

a) $\det\boldsymbol{A} = \begin{vmatrix} 3 & -10 & -10 \\ 0 & 3 & 0 \\ 0 & -5 & -2 \end{vmatrix} = 3\begin{vmatrix} 3 & 0 \\ -5 & -2 \end{vmatrix} = 3(-6) = -18$

b)
$$\begin{array}{rrr|rrr} 3 & -10 & -10 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & -5 & -2 & 0 & 0 & 1 \\ \hline 1 & -\frac{10}{3} & -\frac{10}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & -5 & -2 & 0 & 0 & 1 \\ \hline \end{array} \qquad \begin{array}{rrr|rrr} 1 & -\frac{10}{3} & -\frac{10}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & -2 & 0 & \frac{5}{3} & 1 \\ \hline 1 & -\frac{10}{3} & -\frac{10}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{5}{6} & -\frac{1}{2} \\ \hline \end{array} \qquad \begin{array}{rrr|rrr} 1 & -\frac{10}{3} & 0 & \frac{1}{3} & -\frac{25}{9} & -\frac{5}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{5}{6} & -\frac{1}{2} \\ \hline 1 & 0 & 0 & \frac{1}{3} & -\frac{5}{3} & -\frac{5}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{5}{6} & -\frac{1}{2} \end{array}$$

$$\boldsymbol{A}^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{5}{3} & -\frac{5}{3} \\ 0 & \frac{1}{3} & 0 \\ 0 & -\frac{5}{6} & -\frac{1}{2} \end{pmatrix} = \frac{1}{6}\begin{pmatrix} 2 & -10 & -10 \\ 0 & 2 & 0 \\ 0 & -5 & -3 \end{pmatrix}$$

c) $\begin{vmatrix} 3-\lambda & -10 & -10 \\ 0 & 3-\lambda & 0 \\ 0 & -5 & -2-\lambda \end{vmatrix} = (3-\lambda)\begin{vmatrix} 3-\lambda & 0 \\ -5 & -2-\lambda \end{vmatrix} = (3-\lambda)^2(-2-\lambda) = 0$

$\Longrightarrow \quad \lambda_{1/2} = 3, \lambda_3 = -2$

EV zu $\lambda_{1/2} = 3$:
$$\begin{array}{rrr} 0 & -10 & -10 \\ 0 & 0 & 0 \\ 0 & -5 & -5 \\ \hline 0 & 1 & 1 \end{array}$$

EV zu $\lambda_3 = -2$:
$$\begin{array}{rrr} 5 & -10 & -10 \\ 0 & 5 & 0 \\ 0 & -5 & 0 \\ \hline 1 & -2 & -2 \\ 0 & 1 & 0 \\ \hline 1 & 0 & -2 \\ 0 & 1 & 0 \end{array}$$

$x_2 = -x_3,\ x_1$ beliebig,

EV $C\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + D\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$x_1 = 2x_3,\ x_2 = 0,$ EV $E\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$