Aufgabe 14.13

Die Funktion $f(x) = \begin{cases} \sin|x| & 0 \le |x| \le \frac{\pi}{2} \\ 1 & \frac{\pi}{2} < |x| \le \pi \end{cases}$ werde 2π–periodisch fortgesetzt.

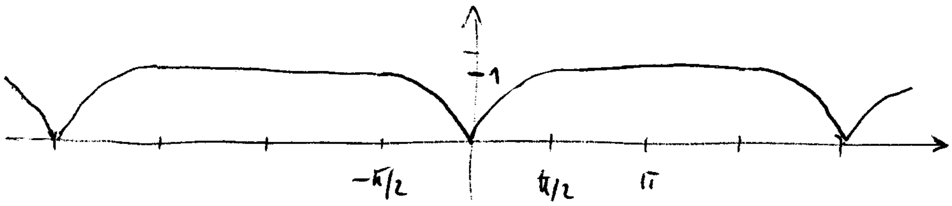
a) Skizzieren Sie die Funktion!

b) Approximieren Sie $f(x)$ mittels Fourierentwicklung durch ein trigonometrisches Polynom 2. Grades $T_2(x) = \frac{a_0}{2} + \sum_{k=1}^{2} a_k \cos kx + b_k \sin kx$!

Hinweis: $\cos\alpha \sin\beta = \frac{1}{2}(\sin(\alpha+\beta) - \sin(\alpha-\beta))$

Lösung:

a)



b) $f(x)$ gerade $\implies$ $b_k = 0$ für alle k

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos kx\,dx = \frac{2}{\pi}\int_0^{\pi} f(x)\cos kx\,dx = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin x \cos kx\,dx + \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} \cos kx\,dx$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \Big(\sin(k+1)x - \sin(k-1)x\Big)\,dx + \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} \cos kx\,dx$$

$$k=0: \quad a_0 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin x\,dx + \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} dx = -\frac{2}{\pi}\cos x\Big|_0^{\frac{\pi}{2}} + \frac{2}{\pi}\,\frac{\pi}{2} = \frac{2}{\pi} + 1$$

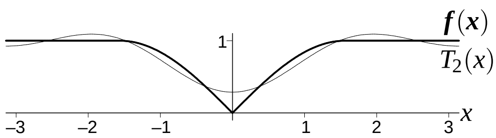
$$k=1: \quad a_1 = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sin 2x\,dx + \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} \cos x\,dx = -\frac{\cos 2x}{2\pi}\Big|_0^{\frac{\pi}{2}} + \frac{2}{\pi}\sin x\Big|_{\frac{\pi}{2}}^{\pi} = -\frac{-1-1}{2\pi} + \frac{2}{\pi}(0-1) = \frac{1}{\pi} - \frac{2}{\pi}$$

$$= -\frac{1}{\pi}$$

$$k=2: \quad a_2 = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} (\sin 3x - \sin x)\,dx + \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} \cos 2x\,dx = -\frac{\cos 3x}{3\pi}\Big|_0^{\frac{\pi}{2}} + \frac{\cos x}{\pi}\Big|_0^{\frac{\pi}{2}} + \frac{\sin 2x}{\pi}\Big|_{\frac{\pi}{2}}^{\pi} = \frac{1}{3\pi} - \frac{1}{\pi}$$

$$= -\frac{2}{3\pi}$$

also $f(x) \approx T_2(x) = \underline{\underline{\frac{1}{\pi} + \frac{1}{2} - \frac{1}{\pi}\cos x - \frac{2}{3\pi}\cos 2x}}$



(Es ginge weiter mit $-\frac{1}{3\pi}\cos 3x - \frac{2}{15\pi}\cos 4x \,\ldots$.)