

Aufgabe 13.104

Berechnen Sie das Integral $\int_0^{\infty} \frac{65}{x^2 + 15x + 50} dx$!

Lösung:

$$x^2 + 15x + 50 = 0, \quad x_{1/2} = -\frac{15}{2} \pm \sqrt{\frac{225}{4} - \frac{200}{4}} = -\frac{15}{2} \pm \frac{5}{2} = \begin{cases} -10 \\ -5 \end{cases}$$

$$\frac{65}{x^2 + 15x + 50} = \frac{A}{x+10} + \frac{B}{x+5}$$

$$65 = A(x+5) + B(x+10) = (A+B)x + 5A + 10B$$

$$A + B = 0$$

$$5A + 10B = 65$$

$$A + 2B = 13, \quad B = 13, \quad A = -13$$

$$\int \frac{65}{x^2 + 15x + 50} dx = 13 \int \frac{dx}{x+5} - 13 \int \frac{dx}{x+10} = 13 \ln \frac{x+5}{x+10} + C$$

$$\int_0^{\infty} \frac{65}{x^2 + 15x + 50} dx = \lim_{A \rightarrow \infty} \int_0^A \frac{65}{x^2 + 15x + 50} dx = 13 \lim_{A \rightarrow \infty} \ln \frac{x+5}{x+10} \Big|_0^A = 13 \left(-\ln \frac{1}{2} \right) = \underline{\underline{13 \ln 2 \approx 9,01}}$$