Aufgabe 13.58

Berechnen Sie die bestimmten Integrale a) $\int\limits_0^4 \frac{\mathrm{d}x}{x^2+16}$ und b) $\int\limits_2^4 \frac{|x-3|}{x^2}\,\mathrm{d}x$!

Lösung:

$$\text{a)}\ \int\limits_0^4 \frac{\mathrm{d}x}{x^2+16} = \frac{1}{16}\int\limits_0^4 \frac{\mathrm{d}x}{\frac{x^2}{16}+1} = \frac{1}{16}\int\limits_0^4 \frac{\mathrm{d}x}{\left(\frac{x}{4}\right)^2+1} = \frac{1}{16}\int\limits_0^1 \frac{4\,\mathrm{d}t}{t^2+1} = \frac{1}{4}\int\limits_0^1 \frac{4\,\mathrm{d}t}{t^2+1} = \frac{1}{4}\arctan t\Big|_0^1$$

$$= \frac{1}{4}(\arctan 1 - \arctan 0) = \frac{1}{4}\left(\frac{\pi}{4}-0\right) = \frac{\pi}{16}$$

$$\text{b)}\ \int\limits_2^4 \frac{|x-3|}{x^2}\,\mathrm{d}x = \int\limits_2^3 \frac{3-x}{x^2}\,\mathrm{d}x + \int\limits_3^4 \frac{x-3}{x^2}\,\mathrm{d}x = \int\limits_2^3 \left(\frac{3}{x^2}-\frac{1}{x}\right)\mathrm{d}x + \int\limits_3^4 \left(\frac{1}{x}-\frac{3}{x^2}\right)\mathrm{d}x$$

$$= \left[-\frac{3}{x}-\ln x\right]_2^3 + \left[\ln x + \frac{3}{x}\right]_3^4 = \left(-1-\ln 3+\frac{3}{2}+\ln 2\right) + \left(\ln 4+\frac{3}{4}-\ln 3-1\right)$$

$$= -2-2\ln 3+\frac{3}{2}+\ln 2+\ln 4+\frac{3}{4} = \frac{1}{4}+\ln\frac{2\cdot 4}{3^2} = \frac{1}{4}+\ln\frac{8}{9} \approx 0.1322$$