Aufgabe 13.57

Berechnen Sie $\displaystyle\int_1^{10}(3x^2+4x+5)\lg x\,dx$!

Lösung:

$$\int(3x^2+4x+5)\lg x\,dx=\int(3x^2+4x+5)\frac{\ln x}{\ln 10}\,dx=\frac{1}{\ln 10}\left((x^3+2x^2+5x)\ln x-\int(x^2+2x+5)\,dx\right)$$

$$=\frac{1}{\ln 10}\left((x^3+2x^2+5x)\ln x-(\frac{x^3}{3}+x^2+5x)\right)$$

$$=(x^3+2x^2+5x)\lg x-\frac{x^3/3+x^2+5x}{\ln 10}$$

$$\int_1^{10}(3x^2+4x+5)\lg x\,dx=(x^3+2x^2+5x)\lg x-\frac{x^3/3+x^2+5x}{\ln 10}\Bigg|_1^{10}$$

$$=1250-\frac{1000/3+150}{\ln 10}+\frac{1/3+6}{\ln 10}=1250-\frac{333+144}{\ln 10}=\underline{\underline{1250-\frac{477}{\ln 10}}}\approx 1042.84$$