Aufgabe 13.48

Berechnen Sie $\int\limits_0^{\frac{\pi}{6}} \sin x \cdot \cos x \,\mathrm{d}x$

a) mittels Integration durch Substitution und
b) durch partielle Integration!

Lösung:

a) Substitution: $t = \sin x, \quad \dfrac{\mathrm{d}t}{\mathrm{d}x} = \cos x, \quad \mathrm{d}t = \cos x \,\mathrm{d}x$

$$\int\limits_0^{\frac{\pi}{6}} \sin x \cdot \cos x \,\mathrm{d}x = \int\limits_0^{\frac{1}{2}} t \,\mathrm{d}t = \frac{t^2}{2}\bigg|_0^{\frac{1}{2}} = \frac{1}{8} - 0 = \underline{\underline{\frac{1}{8}}}$$

b) $$\int\limits_a^b u(x)\, v'(x)\,\mathrm{d}x = u(x)\, v(x)\Big|_a^b - \int\limits_a^b u'(x)\, v(x)\,\mathrm{d}x$$

$$\int\limits_0^{\frac{\pi}{6}} \sin x \cos x \,\mathrm{d}x = \sin x \sin x\Big|_0^{\frac{\pi}{6}} - \int\limits_0^{\frac{\pi}{6}} \cos x \sin x \,\mathrm{d}x = \frac{1}{4} - \int\limits_0^{\frac{\pi}{6}} \cos x \sin x \,\mathrm{d}x$$

$$2\int\limits_0^{\frac{\pi}{6}} \sin x \cos x \,\mathrm{d}x = \frac{1}{4}, \quad \int\limits_0^{\frac{\pi}{6}} \sin x \cos x \,\mathrm{d}x = \underline{\underline{\frac{1}{8}}}$$