Aufgabe 13.35

Berechnen Sie $\displaystyle\int \frac{4x^2+3x+1}{x^3-x^2+x-1}\,\mathrm{d}x$!

Lösung:

$$\frac{4x^2+3x+1}{x^3-x^2+x-1}=\frac{4x^2+3x+1}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$4x^2+3x+1=A(x^2+1)+(Bx+C)(x-1)=Ax^2+A+Bx^2+Cx-Bx-B=(A+B)x^2+(C-B)x+A-C$$

$$x^2:\quad 4=A+B \qquad\qquad 4=C+1+C-3=2C-2,\ \ C=3,\ \ A=4,\ \ B=0$$
$$x:\quad 3=C-B \qquad B=C-3$$
$$1:\quad 1=A+C \qquad A=C+1$$

$$\frac{4x^2+3x+1}{x^3-x^2+x-1}=\frac{4}{x-1}+\frac{3}{x^2+1}$$

$$\int \frac{4x^2+3x+1}{x^3-x^2+x-1}\,\mathrm{d}x=4\int \frac{\mathrm{d}x}{x-1}+3\int \frac{\mathrm{d}x}{x^2+1}=4\ln|x-1|+3\arctan x+C$$