

Aufgabe 13.32

Berechnen Sie durch Partialbruchzerlegung:

$$\begin{aligned} \text{a) } & \int \frac{2x^3 - x^2 - 10x + 19}{x^2 + x - 6} dx, & \text{b) } & \int \frac{9x^2 - 2x - 8}{x^3 - 4x} dx, & \text{c) } & \int \frac{3x^2 + 7x + 1}{x^3 + 2x^2 + x} dx, \\ \text{d) } & \int \frac{2x^3 + 5x^2 + 27x + 12}{x^2 + 2x + 10} dx, & \text{e) } & \int \frac{x^4 - 5x^3 + 7x^2 - 13x - 10}{x^3 - 5x^2} dx ! \end{aligned}$$

Lösung:

$$\text{a) } (2x^3 - x^2 - 10x + 19) : (x^2 + x - 6) = 2x - 3 + \frac{5x + 1}{x^2 + x - 6},$$

$$x^2 + x - 6 = 0 \text{ für } x_1 = 2, x_2 = -3,$$

$$\frac{2x^3 - x^2 - 10x + 19}{x^2 + x - 6} = 2x - 3 + \frac{5x + 1}{x^2 + x - 6} = 2x - 3 + \frac{A}{x - 2} + \frac{B}{x + 3},$$

$$5x + 1 = A(x + 3) + B(x - 2) = (A + B)x + (3A - 2B), \quad A + B = 5, \quad 3A - 2B = 1,$$

$$A = \frac{11}{5}, \quad B = \frac{14}{5},$$

$$\begin{aligned} \int \frac{2x^3 - x^2 - 10x + 19}{x^2 + x - 6} dx &= \int (2x - 3) dx + \frac{11}{5} \int \frac{dx}{x - 2} + \frac{14}{5} \int \frac{dx}{x + 3} \\ &= x^2 - 3x + \frac{11}{5} \ln|x - 2| + \frac{14}{5} \ln|x + 3| + C \end{aligned}$$

$$\text{b) } \int \frac{9x^2 - 2x - 8}{x^3 - 4x} dx = \int \left(\frac{2}{x} + \frac{3}{x - 2} + \frac{4}{x + 2} \right) dx$$

$$= 2 \ln|x| + 3 \ln|x - 2| + 4 \ln|x + 2| + \ln C = \ln(Cx^2|x - 2|^3|x + 2|^4)$$

$$\text{c) } \int \frac{3x^2 + 7x + 1}{x^3 + 2x^2 + x} dx = \int \left(\frac{1}{x} + \frac{2}{x + 1} + \frac{3}{(x + 1)^2} \right) dx$$

$$= \ln|x| + 2 \ln|x + 1| - 3 \frac{1}{x + 1} + \ln C = \ln(C|x|(x + 1)^2) - 3 \frac{1}{x + 1}$$

$$\text{d) } \int \frac{2x^3 + 5x^2 + 27x + 12}{x^2 + 2x + 10} dx = \int \left(2x + 1 + \frac{5x + 2}{x^2 + 2x + 10} \right) dx$$

$$= \int (2x + 1) dx + \frac{5}{2} \int \frac{d(x^2 + 2x + 10)}{x^2 + 2x + 10} - 3 \int \frac{dx}{x^2 + 2x + 10}$$

$$= \int (2x + 1) dx + \frac{5}{2} \int \frac{d(x^2 + 2x + 10)}{x^2 + 2x + 10} - \frac{3}{3} \int \frac{d\frac{x+1}{3}}{\left(\frac{x+1}{3}\right)^2 + 1}$$

$$= x^2 + x + \frac{5}{2} \ln(x^2 + 2x + 10) - \arctan \frac{x + 1}{3} + C$$

$$\text{e) } (x^4 - 5x^3 + 7x^2 - 13x - 10) : (x^3 - 5x^2) = x + \frac{7x^2 - 13x - 10}{x^3 - 5x^2} = x + \frac{7x^2 - 13x - 10}{x^2(x - 5)},$$

$$\frac{7x^2 - 13x - 10}{x^2(x - 5)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 5},$$

$$7x^2 - 13x - 10 = Ax(x-5) + B(x-5) + Cx^2 = (A+C)x^2 + (-5A+B)x - 5B,$$

$$-5B = -10 \Rightarrow B = 2, \quad -5A+B = -13 \Rightarrow A = 3, \quad A+C = 7 \Rightarrow C = 4,$$

$$\begin{aligned} \int \frac{x^4 - 5x^3 + 7x^2 - 13x - 10}{x^3 - 5x^2} dx &= \int x dx + 3 \int \frac{dx}{x} + 2 \int \frac{dx}{x^2} + 4 \int \frac{dx}{x-5} \\ &= \frac{x^2}{2} + 3 \ln|x| - \frac{2}{x} + 4 \ln|x-5| + D = \frac{x^2}{2} - \frac{2}{x} + \ln C(x-5)^4 |x|^3 \end{aligned}$$